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Safety Analysis of Composite Materials for Existing and New Construction

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SAFETY ANALYSIS OF COMPOSITE MATERIALS FOR EXISTING AND NEW CONSTRUCTION

By

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In four studies, the live load factors for the design of reinforced concrete (RC) structures, and the strength reduction factors assigned to the elements that use FRP material as internal or external reinforcement, are reevaluated against the values in current practice.

Taking advantage of the theory of the reliability of structures, Studies I and II incorporate the life-time into the live load factor of an RC element. To this end, a statistical model is established upon the recognized axioms about the probabilistic distributions of load and resistance and that the live load factor of 1.60, stated by the current building code for RC structures, may account for the variations of the live load in a period of 50 years. The outcome of these studies describes the live load factor as ascending functions of life-time, which meet the predetermined value of 1.60 for a life-time of 50 years. The same formulation also provides a solution to the problem of the effect of the under or over-design on the expected life-time of a member.

Studies III and IV also employ the theory of reliability, but this time to calibrate the strength reduction factors of the elements that use FRP reinforcement. Study III concentrates on flexural members with internal FRP reinforcement, while the subject of Study IV is externally strengthened flexural members with the focus on near-surface-

mounted (NSM) FRP bars. The novelty of Study III is the introduction of a new approach to the calibration of reduction factors which is referred to as the "comparative reliability". The current North American guidelines that regulate the design of the RC members internally reinforced with FRP bars, derive their factors by targeting preset levels of reliability that are, sometimes, not even achievable by the ordinary steel reinforced concrete members. Conceding that the latter elements are of sufficient safety, the comparative reliability is a method to calculate the strength reduction factors of the newly introduced elements in harmony with the old ones. This approach to strength reduction factors minimizes the penalizing of one material in favor of the other, while maintaining a uniform level of safety for all.

Unlike Study III, that uses a database of experimental results to obtain its essential statistical input, Study IV generates its own database of externally strengthened RC beams and slabs with NSM FRP bars, by benefiting from simulation techniques. The combination of the computerized simulation and comparative reliability creates an original approach to the calibration of the strength reduction factors of NSM systems, while in the current guidelines, the reduction factors are selected by judgment and consensus and lack a theoretical and experimental foundation. Furthermore, this study eliminates the current partial strength reduction factor, assigned by the current design guideline to FRP contribution, and achieves an inclusive factor for NSM FRP systems.

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CHAPTER 1

1. INTRODUCTION

1.1 PREFACE

The two design parameters that determine the safety of a structural member are the load and strength reduction factors. For common structural materials, such as steel and reinforced concrete, these parameters are very well established, while for the newly introduced structural composites these factors are still in need of validation as:

The load factors are designed to account for the uncertainties relative to loads and therefore, it may be argued that, they are material-independent. Nevertheless, when the strengthening of an existing structure is concerned, the expected remaining life-time can be different, probably shorter, from what is expected from new construction. A shortened life-time, certainly, limits the changeability of the time-dependent loads such as live, wind and seismic loads. The first two studies of this thesis try to reflect this time-induced change of randomness in the special case of the live load factor. The proposed method, however, is expandable to cover other loads as wind, earthquake and snow.

The strength reduction factors, which are supposed to temper the randomness of resistance, are undoubtedly material-dependent. In this aspect, the current North American guidelines developed by the American Concrete Institute (ACI), that control the design of the structural members that use composites (also known as fiber-reinforced polymers or FRPs), either internally or externally, suffer from two obvious shortcomings. They either derive these factors by imposing stringent and non-flexible safety measures on the members they regulate, as is the case for the internally FRP reinforced flexural RC members, or merely replicate the factors assigned to the ordinary RC elements, with the addition of partial reduction factors for the FRP contribution, as is the case for the externally strengthened RC members. Again, this thesis, in its two last studies, is an attempt to calibrate the strength reduction factors, so that neither the competitiveness nor the safety of the elements with composite materials is compromised. Study III deals with the flexural and shear strength reduction factors assigned to beams and slabs reinforced internally with FRP bars. Study IV calibrates the flexural reduction factors for external strengthening with FRP bars in the special case of near-surface-mounted (NSM) systems.

The reliability analysis is the recurring theme in all the four studies that form this thesis. The reliability analysis can simply be described as the assessment of the probability of failure of a certain element subject to a certain load. If the statistical parameters of load and resistance are known or estimated, and if a desired level of safety or equally an acceptable probability of failure, is determined, the strength and load factors can be calculated by reliability-based-criteria. Since the field of the reliability of structures is still fresh and not fully explored, this thesis can also make an original contribution to the advancement of the subject matter which is no less important than its obvious goals stated earlier.

1.2 OBJECTIVES AND OUTLINE

The thesis may be outlined by **Table 1.1** which summarizes the abovementioned points and the intended improvements. The four studies share similarities in their general direction and goals; however, each of them possesses a distinctive character that separates it from the rest:

- Study I achieves its goal with analytical means that benefit from simplifying assumptions.
- Study II uses a numerical method to enhance the accuracy of the analytical results of Study I.
- Study III prepares the theoretical ground of calibrating the strength reduction factors and applies it to a set of experimental data.
- Study IV follows the calibration concept of Study III, but gains its distinction from generating its input database by computerized simulation.

The similarities provide a smooth transition from one study to the other, while the differences make each study maintain its independent identity.

Table 1.1: Outline of the objectives

Study	**Subject of study**	**Parameter of concern**	**According to ACI**	**Objective of study**
I and II	ACI 318-08 RC elements	Live load factor	$\gamma_L = 1.60$ [(1)]	$\gamma_{L_n} = f(n)$ [(2)]
III	ACI 440.1R-06 FRP RC elements	Strength reduction factors	Flexural factors are calibrated so that: β(FRP RC)≥3.5 [(4)(5)] Shear factors are based on ACI 318-08 [(6)], with no reliability analysis performed.	Calibrate the reduction factors using the reliability analysis so that: β(FRP RC)≈β(Steel RC) β(NSM)≈β(Steel RC) [(4)]
IV	ACI 440.2R-08 strengthened RC elements	Strength reduction factor for flexural NSM systems	Flexural factors are based on ACI 318-08 [(7)], with the addition of a partial factor, ψ_f=0.85, for FRP [(8)]. No reliability analysis is performed.	

[(1)] ACI 318-08:9.2.1
[(2)] n is the expected life-time (years).
[(3)] ACI 440.1R-06:8.2.3
[(4)] β is the reliability index.
[(5)] ACI 440.1R-06:8.2.3
[(6)] ACI 440.1R-06:9.1.1
[(7)] ACI 440.2R-08:10.2.7
[(8)] ACI 440.2R-08:10.2.10

1.3 NOTATIONS

A_f= area of FRP reinforcement, in.^{2} (mm^2)

A_{fv}= amount of FRP shear reinforcement within spacing s, in.^{2} (mm^2)

A_s= area of steel reinforcement, in.^{2} (mm^2)

b= width of the beam or slab strip, in. (mm)

C_E= environmental reduction factor

c= concrete cover to the centroid of steel reinforcement, in. (mm)

D= dead loads, or related internal moments and forces

d= dimensionless dead load (random variable)

d^*= value of d at design point

d_b= diameter of reinforcing bar, in. (mm)

d_f= effective depth of tensile FRP reinforcement, in. (mm)

d_s= effective depth of tensile steel reinforcement, in. (mm)

E_c= modulus of elasticity of concrete according to ACI 318-11, ksi (GPa)

E_f= modulus of elasticity of FRP bars, ksi (GPa)

E_s= modulus of elasticity of steel bars, ksi (GPa)

$F_X(x)$= cumulative distribution function (CDF) for a random variable X

f= ratio of stress in FRP to its debonding stress

f'_c= specified compressive strength of concrete, ksi (MPa)

f_f= stress level in FRP reinforcement, psi (MPa)

f_{fb}= strength of bent portion of FRP bar, ksi (MPa)

f_{fd}= design stress of externally bonded FRP reinforcement, ksi (MPa)

f_{fu}= ultimate longitudinal tensile strength of FRP bars, ksi (MPa)

f^*_{fu}=ultimate tensile strength of the FRP material as reported by the manufacturer, ksi (MPa)

f_{fv} =tensile strength of FRP for shear design, ksi (MPa)

$f_X(x)$= probability density function (PDF) for a random variable X

f_y= yield strength of steel reinforcement, ksi (MPa)

G= limit state function

g= normal comparative limit state function

g_0= a specific value of g

k= gamma distribution parameter

k_c= ratio of depth of neutral axis to reinforcement depth

L= live loads, or related internal moments and forces

L'= nominal live load capacity of a member

l= dimensionless live load (random variable)

l^*= value of l at design point

N= total number of sets of random samples

N_f= number of failures

P= estimated probability of failure

P_{true}= correct probability of failure

$P(X \leq x)$ = probability of the event $X \leq x$

p= probability of the event $g<g_0$

p_c= probability of the event $g<0$

Q= total load (random variable)

Q_D= dead load (random variable)

Q_L= live load (random variable)

Q_N= nominal value of total load, or related internal moments and forces

q= dimensionless total load (random variable)

q^*= value of q at design point

M_f= contribution of FRP to flexural resistance disregarding professional factor

M_n= nominal flexural strength, kip.in. (kN.m)

M_{nf}= contribution of FRP reinforcement to nominal flexural strength, kip.in. (kN.m)

M_{ns}= contribution of steel reinforcement to nominal flexural strength, kip.in. (kN.m)

M_s= contribution of steel to flexural resistance disregarding professional factor

N_i= nominal value of resistance of ith element (i=1, 2)

n_f= ratio of modulus of elasticity of FRP bars to modulus of elasticity of concrete

n= life–time (years)

P_f= professional factor for FRP contribution

P_s= professional factor for steel contribution

R= resistance (random variable)

R_f= contribution of FRP to total flexural resistance (random variable)

R_i= resistance of ith element (i=1, 2)

R_N= nominal capacity of an element

R_s= contribution of steel to total flexural resistance (random variable)

r= dimensionless resistance (random variable)

r^*= value of r at design point

r_b= internal radius of bend in FRP reinforcement, in. (mm)

s= stirrup spacing, in. (mm)

U= required strength of an element (n=50)

U_n= required strength of an element (life-time of n years)

u= extreme value distribution (EVD) Type I parameter

u_i= ith sample of a uniformly distributed variable between 0 and 1

u_n = EVD Type I parameter for live load in a life-time of n years

V_c= nominal shear strength provided by concrete, kips (kN)

V_f= shear resistance provided by FRP stirrups, kips (kN)

V_n= nominal shear strength at section, kips (kN)

W= nominal value of wind load

x^*= design point

x_i=ith sample of a random variable X

z^*= reduced design point

z_x= reduced form of the random variable X

z_x^*= value of z_x at design point

α = extreme value distribution (EVD) Type I parameter

α_1= multiplier on f'_c to for an equivalent rectangular stress distribution for concrete

α_n = EVD Type I parameter for live load in a life-time of n years

β= reliability index

β_1 = ratio of depth of equivalent rectangular stress block to depth of the neutral axis

β_n = reliability index for a life-time of n years

β_T= target reliability index

$\Gamma(k)$= gamma function

γ= total load factor for a life-time of 50 years

γ_D= dead load factor per ACI 318-11 (1.2)

γ_L = live load factor per ACI 318-11 (1.6)

γ_{L_n} = live load factor for a life-time of n years

γ_n = total load factor for a life-time of n years

Δ= strengthening level

δ_{L_n} = coefficient of variation of live load for a life-time of n years

δ_{Q_n} = coefficient of variation of total load for a life-time of n years

ε_{bi}= strain level in concrete substrate at time of FRP installation, in./in. (mm/mm)

ε_{cu}=0.003 in./in. (mm/mm), ultimate axial strain of concrete

ε_i= intermediary sepeation variables (i=1, 2, c)

ε_n = intermediary separation variable for a life-time of n years

ε_s= strain in steel reinforcement, in./in. (mm/mm)

ε_{sy}= strain corresponding to yield strength of steel reinforcement, in./in. (mm/mm)

δ_X = coefficient of variation of a random variable X

θ= gamma distribution parameter

κ= life-time modification coefficient

κ_m=0.70, dimensionless bond-dependent coefficient for flexure

λ_{L_n} = bias factor of live load for a life-time of n years

λ_{Q_n}= bias factor of total load for a life-time of n years

λ_X = bias factor of a random variable X

μ= intermediary variable

μ_{L_n} = mean value of live load for a life-time of n years

μ_X = mean value of a random variable X

μ_X^e= equivalent normal mean value of a random variable X

ρ= ratio of live load to total load

ρ_{fl}= FRP longitudinal reinforcement ratio

ρ_{fv}= FRP transverse reinforcement ratio

ρ_s =steel reinforcement ratio

σ_X= standard deviation of a random variable X

σ_X^e = equivalent normal standard deviation of a random variable X

Φ= cumulative distribution function (CDF) for standard normal distribution

ϕ= probability density function (PDF) for standard normal distribution

Ø= strength reduction factor

ψ_f=0.85 partial reduction factor for FRP

ω_b= overall reinforcement index producing balanced strain conditions

ω_f= reinforcement index for FRP bars

ω_{fb}= FRP reinforcement index producing balanced strain conditions

$\omega_{s=}$ reinforcement index for steel bars

ω_{sb}= steel reinforcement index producing balanced strain conditions

CHAPTER 2

2. STUDY I: INCORPORATING EXPECTED LIFE-TIME INTO LIVE LOAD FACTOR FOR RC STRUCTURES USING RELIABILITY ANALYSIS

2.1 BACKGROUND

Current design codes are mostly silent about the expected life-time of a structure, but it is widely believed and agreed upon that their provisions are intended for a life-time of at least 50 years (Nowak and Collins 2000, Ellingwood and Galambos 1982, McCormac 1989). The design live load proposed by ASCE 7-10 is by definition the maximum live load that is predicted to be experienced by an element during its life-time, whereas normally only a fraction of this maximum, known as the sustained live load, is applied to the element. Infrequent but typically drastic additions to the sustained live load are categorized as transient live load, a time-dependent random variable. Therefore, the design live load is describable as the maximum value of the live load, sustained plus transient, over the expected life-time of 50 years. Noting that sustained live load is also a time dependent random variable, the maximum value of live load over a time range (design live load for that time range) is another random variable whose statistical properties naturally approach those of the sustained live load as the life-time shortens. When the life time reaches its minimum, in other words when it is reduced to a given point in time, these two become identical, thus sustained live load is often called arbitrary-point-in-time (APT) live load. With such a definition of design live load, it is a foregone conclusion that, if the life-span of a structure is shortened or prolonged, the design live load has to be affected in the same manner (i.e., is reduced or increased, respectively). As a result, when the life-span is shorter than 50 years, in such cases as a new but temporary building, or an existing building being repaired with the aim of completing its remaining life-time, a certain degree of relaxation to the code's live load factor is logical. Nonetheless, the question of calculation method for this reduction in load factor (or increase in case of a life-time of more than 50 years) remains unanswered. The same question may be posed in a somewhat more tangible format, that is: how does over-design (or under-design) prolong (or shorten) the expected life-time from what the code intended (presumably 50 years)?

This study is an attempt to tackle these questions using a method of reliability analysis supported by test data and results available in the technical literature. This is, however, only a load-oriented solution the consequences of which cannot be overstated inasmuch as other factors (most importantly environmental effects) are not accounted for. To be more precise, as the changeability of the live load is reflected in its factor, the possible decline of the nominal strength or the deterioration of the structure by time may be mirrored in the strength reduction factor, thus decoupling the problem. The latter, however, lies outside the scope of this study which only deals with the variability of the live load with time. Still, by covering the aspect of the problem dealing with the variation of live load with time, this chapter provides a partial solution to the larger question. Attention should also be paid to the definition of life-time when the natural deterioration of the structure is overlooked. In this sense, life-time is not a time span after which the structure is deemed useless, but is a period during which the probability of failure is confined within the limits tolerated by the design code.

In this chapter a method of reliability analysis is utilized to calibrate the live load factor based on the expected life-time of a cast-in-place reinforced concrete (RC) structure. The probabilistic parameters of live load are computed for different life-times and live load factors are then suggested so that the same level of safety is maintained for any given life-span. An example is presented that demonstrates how modified live load factors can be conducive to making decision on the necessity and the level of the required strengthening in a concrete structure in need of repair.

2.2 OUTLINE

The components of this study that are also sequentially presented include:

1) Collection of data (resistance and load models):

- Statistical parameters (i.e., mean and standard deviation) of resistance for each element and its (ultimate) limit state are gathered from the literature (Nowak and Szerszen 2003).

- Dead load and live load are assumed to be the only loads exerted on an element and are combined according to the relevant load combination of ASCE 7 (ultimate strength design) which is concurrent with ACI 318-11 (U=1.2D+1.6L). When only live and dead load are of concern, another possible combination is U=1.4D, which is only critical if the live load is minimal. Since the focus of this sudy is on the live load, this combination is neglected.
- Statistical parameters of dead and live load for both cases of a 50-year period and arbitrary-point-in-time are gathered from the literature (Nowak and Szerszen 2003 and Ellingwood and Galambos 1982).

These brief sections obtained from existing studies prepare the input for the following sections which make up the original contribution of this work to the existing body of knowledge.

2) Process of data:

- Based on these data, a probabilistic model is constructed that permits the computation of the statistical parameters of live load as functions of life-time. This part provides the data required for the calibration process.

3) Calibration procedure:

- Limit state functions are defined according to the ultimate load-carrying capacity of the element in question. The limit states are defined as exceeding: a) the ultimate moment carrying capacity for flexural members (RC beams and slabs); b) the ultimate shear capacity of RC beams; and, c) the ultimate compressive capacity of tied RC columns (concrete crushing). Thus the terms “limit state” and “failure mode” might be used interchangeably.
- The reliability index for a 50-year life-span is calculated according to ACI 318-11 load and safety factors together with relevant statistical parameters of live and dead load. This reliability index serves as the “target reliability” or the safety level provided by the design code.
- For each life-time, the live load factor, γ_{L_n}, is calculated so that the target reliability index is achieved.
- Life-time modification coefficient, κ, is then introduced to unify the formulation of the live load factors. κ is calculated for several cases that can cover a range of loadings and limit states.

By describing the live load factor, γ_L, as a function of the expected life-time of a structure, this chapter tries to add another dimension to the load and resistance factor (ultimate strength) design method of ACI 318-11. This approach offers the clear advantage of an economical yet safe design when the expected life-time is different from what the code intended. Additionally, it provides a methodology to assess the expected life of structures in need of strengthening, which is a critical factor in deciding upon the urgency and level of the required repair.

2.3 RESISTANCE MODEL

The structural elements considered in this chapter are cast-in-place RC flexural members (beams and slabs with tension-controlled failure) and cast-in-place RC compression members (axially loaded tied columns with compression-controlled failure). The ultimate limit states are flexural moment capacity for beams and slabs, shear capacity for beams, and compressive capacity for columns. Materials utilized in construction are ordinary concrete and steel reinforcing bars. The construction method is cast-in-place. For such members and conditions, the statistical parameters of resistance, R, are taken from Nowak and Szerszen (2003) and shown in **Table 2.1**. These parameters include: 1) bias factor or λ (the ratio of mean to the nominal or design value of a random variable); 2) coefficient of variation (CoV) or δ; and, 3) the probabilistic distribution of the variable.

2.4 LOAD MODEL

The two most common load components, dead and live loads, are herein considered. The statistical parameters and properties of dead and live loads are taken from the available literature (Nowak and Szerszen 2003, Ellingwood and Galambos 1982) and summarized in **Table 2.2** with its parameters defined similar to **Table 2.1**. Two sets of these parameters are introduced: the first set, arbitrary-point-in-time (APT), relates to the load expected to act on the structure at any given time; while the second set corresponds to the maximum load expected in the 50-year life-time of the structure. Dead load is basically time independent, therefore the two sets (i.e., APT and 50-year) are identical. In addition to time, live load parameters are also functions of the influence area so that by increasing that area, the live load tends to grow more deterministic and less random. Here, an influence area of 430 sq. ft. (40 m^2) is considered as a reasonable assumption which also corresponds to the values in **Table 2.2**. This concludes the collection of data from literature. The following section details the utilization of the statistical modelling in order to interpolate (for life-times shorter than 50 years) and extrapolate (for life-times longer than 50 years) these parameters and hence generate the essential data for the calibration of the live load factor.

2.5 CALCULATION OF STATISTICAL PARAMETERS OF LIVE LOAD AS FUNCTIONS OF LIFE-TIME

Building upon **Table 2.2**, this study tries to project the statistical parameters of live load, λ_L and δ_L, for different life-spans.

The probabilistic nature of maximum live load in a certain period of time is assumed to be characterized by extreme value distribution (EVD) Type I. Such a distribution is defined by its cumulative distribution function (CDF), $F_X(x)$, and probability density function (PDF), $f_X(x)$, as shown in Equations 2.1 and 2.2 (Nowak and Collins 2000):

$$F_X(x) = P(X \leq x) = e^{-e^{-\alpha(x-u)}} \tag{2.1}$$

$$f_X(x) = \alpha e^{-e^{-\alpha(x-u)}} e^{-\alpha(x-u)} \tag{2.2}$$

In Equations 2.1 and 2.2, X is a random variable and $P(X \leq x)$ denotes the probability of X being equal or smaller than a specific value of x. Parameters α and u are related to mean, μ_X, and standard deviation, σ_X, of X by (Haldar and Mahadevan 2000):

$$\mu_X = u + \frac{0.5772}{\alpha} \tag{2.3}$$

$$\sigma_X = \frac{\pi}{\sqrt{6}\alpha} \tag{2.4}$$

In order to derive λ and δ for a period of 25 years, it can be argued that a life-time of 50 years can be split into two consecutive 25-year periods. It is assumed that for each sub-life-time, the live load, Q_L, conforms to a Type I distribution. Let $F_{25,Q_L}(l)$, defined by parameters α_{25} and u_{25}, stand for the CDF of live load during each period of 25 years, whereas $F_{50,Q_L}(l)$, representing the CDF over the total life-time, is defined by α_{50} and u_{50} or:

$$\mu_{L_n} = u_n + \frac{0.5772}{\alpha_n} \tag{2.5}$$

$$\sigma_{L_n} = \frac{\pi}{\sqrt{6}\alpha_n} \tag{2.6}$$

Where n is the life-time, i.e., 25 or 50 years in this case. According to Equation 2.1:

$$P(Q_L \leq l)|_{\text{in 50 years}} = F_{50,Q_L}(l) \tag{2.7}$$

$$P(Q_L \leq l)|_{\text{in the first 25 years}} = F_{25,Q_L}(l) \tag{2.8}$$

$$P(Q_L \leq l)|_{\text{in the second 25 years}} = F_{25,Q_L}(l) \tag{2.9}$$

The probability of occurrence of the event in Equation 2.7 is conditional upon occurrence of both events in Equations 2.8 and 2.9 or:

$$P(Q_L \leq l)|_{\text{in 50 years}} = P\left[(Q_L \leq l)|_{\text{in the first 25 years}} \text{ AND } (Q_L \leq l)|_{\text{in the second 25 years}}\right] \tag{2.10}$$

It is justifiable to assume that the loadings in the two periods are uncorrelated and therefore:

$$P(Q_L \leq l)|_{\text{in 50 years}} = \left[P(Q_L \leq l)|_{\text{in the first 25 years}}\right]\left[P(Q_L \leq l)|_{\text{in the second 25 years}}\right] \tag{2.11}$$

Substituting from Equations 2.7 to 2.9 into Equation 2.11:

$$F_{50,Q_L}(l) = F^2_{25,Q_L}(l) \tag{2.12}$$

Hence:

$$e^{-e^{-\alpha_{50}(l-u_{50})}} = e^{-e^{-\alpha_{25}(l-u_{25})+\ln(2)}} \tag{2.13}$$

Which necessitates:

$$\alpha_{50} = \alpha_{25} \tag{2.14}$$

And therefore:

$$\sigma_{L_{50}} = \sigma_{L_{25}} \tag{2.15}$$

And:

$$\alpha_{50}u_{50} - \alpha_{25}u_{25} = \ln(2) \tag{2.16}$$

Hence, recalling Equations 2.5 and 2.6:

$$\mu_{L_{25}} = \mu_{L_{50}} - \frac{\ln(2)}{\alpha_{50}} = \mu_{L_{50}} - \frac{\sqrt{6}}{\pi}\ln(2)\sigma_{L_{50}} = \mu_{L_{50}}\left[1 - \frac{\sqrt{6}}{\pi}\ln(2)\delta_{L_{50}}\right] \tag{2.17}$$

In Equation 2.17, $\delta_{L_{50}}$=0.18, as per **Table 2.2**. In general, for any life-time of *n* years for which a valid assumption of EVD Type I distribution can be made:

$$\mu_{L_n} = \mu_{L_{50}}\left[1 + 0.140\ln\left(\frac{n}{50}\right)\right] \tag{2.18}$$

$$\sigma_{L_n} = \sigma_{L_{50}} \tag{2.19}$$

Note that for *n*=25, Equations 2.18 and 2.19 are identical to Equations 2.17 and 2.15. λ_{L_n}, the bias factor of the live load for a period of *n* years (e.g., $\lambda_{L_{50}}$=1.00 from **Table 2.2**), and δ_{L_n}, the coefficient of variation of the live load for the same period, are defined as:

$$\lambda_{L_n} = \frac{\mu_{Ln}}{L} \tag{2.20}$$

$$\delta_{L_n} = \frac{\sigma_{Ln}}{\mu_{Ln}} \tag{2.21}$$

In Equation 2.20, *L* is the nominal (design) value of live load per code. Eventually, by substituting from Equation 2.20 into 2.18:

$$\lambda_{L_n} = \lambda_{L_{50}}\left[1 + 0.140\ln\left(\frac{n}{50}\right)\right] = 1 + 0.140\ln\left(\frac{n}{50}\right) \tag{2.22}$$

Combining Equations 2.19 and 2.21 results in:

$$\mu_{L_n}\delta_{L_n} = \mu_{L_{50}}\delta_{L_{50}} \tag{2.23}$$

Substituting from Equation 2.20 into Equation 2.23:

$$\lambda_{L_n}\delta_{L_n} = \lambda_{L_{50}}\delta_{L_{50}} = 0.180 \tag{2.24}$$

Or:

$$\delta_{L_n} = \frac{0.180}{1+0.140\ln\left(\frac{n}{50}\right)} \tag{2.25}$$

One must, however, be mindful of the limitations of Equations 2.22 and 2.25. As *n* decreases, the probabilistic distribution of live load deviates from EVD Type I and approaches gamma distribution, which is the distribution corresponding to APT. Therefore, these formulae lose their applicability for small values of *n*. The minimum value of *n* that can justify an EVD Type I distribution is mostly a matter of engineering judgment; nevertheless, mathematically speaking, in order to maintain consistency, it is necessary but may not be sufficient, that *n* be selected so that $\lambda_{L_n} \geq \lambda_{L_{APT}}$=0.24 and $\delta_{L_n} \leq$

$\delta_{L_{APT}}$=0.65, which can be conservatively translated into $n \geq 1$. By imposing this restriction, Equations 2.22 and 2.25 provide a smooth transition between the two cases of point-in-time and 50-year period and even beyond (**Fig. 2.1**). **Table 2.3** also presents the statistical parameters (bias and CoV) for a few selected life-times. In **Fig. 2.1** the origin, n=0, corresponds to APT in **Table 2.3**. The segments of the two curves confined between APT and n=1 are of very little practical interest and at any rate can be obtained accurately enough by a linear interpolation between the two points.

2.6 CALIBRATION OF LIVE LOAD FACTOR BY MEANS OF RELIABILITY ANALYSIS

The safety level of a structural element is measured in terms of reliability index β which is defined as:

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \quad (2.26)$$

In which the subindices R and Q indicate resistance and load respectively. In case of live load, as the mean value of load, μ_{L_n}, is a function of life-time (Equation 2.18) so is the reliability index, unless resistance is adjusted so that a constant level of safety for any given life-time is attained. This constant level or target reliability, β_T, is benchmarked against the already established level of safety of the code:

$$\beta_T = \beta_{50} \quad (2.27)$$

β_{50} is the reliability index for a life-time of 50 years assuming that load and safety factors given by ACI 318-11 pertain to that time span. It is essential that attention be paid to this definition of the target reliability that differs from the conventional target reliability described as a constant value. By this definition, β_T is a function of the relative magnitude of live and dead loads. β_T is also, as the design code intends, a function of the consequence of failure with the more critical elements or failure modes having larger target reliability indices. These variations of the target reliability are quantified in the upcoming sections of this study.

Adjustment of resistance can be implemented by changing design requirements (i.e., the design live load factor, the load magnitude or the strength reduction factor). In this study, this adjustment is applied to the live load factor and is referred to as the calibration of live load factor.

Calculation of target reliability index (β50)

The first step of calibration is the calculation of the reliability index in terms of safety and load factors and statistical parameters of load and resistance. ACI 318-11 requires that the following equation between the nominal values of resistance, R_N , dead load, D, and live load, L, be upheld:

$$\phi R_N = \gamma_D D + \gamma_L L \tag{2.28}$$

In which γ_D=1.2 is the dead load factor, γ_L=1.6 is the live load factor and Ø is the strength reduction factor that assumes different values based on the limit state (**Table 2.4)**.

For RC members complying with this load combination, **Fig. 2.2** displays the target reliability over the whole range of ρ, the ratio of live (L) to total load (D+L). **Table 2.4** also shows the exact value of the index for a few select values of ρ. **Appendix A** provides details of the calculations undertaken to generate the relationship between the reliability index and ρ. Furthermore, **Table 2.4** and **Fig. 2.2** show how target reliability is in agreement with the importance hierarchy of the elements and the suddenness of failure. The highest value of β belongs to columns, the lowest to slabs while beams are placed in between. Again for beams, failure against shear has to satisfy higher levels of reliability compared to the more ductile and less sudden flexural failure.

Modification of live load factor

ACI 318-11 load-resistance requirement (Equation 2.28) can be generalized for any life-span of *n* years as:

$$\phi R_N = \gamma_D D + \gamma_{L_n} L \tag{2.29}$$

The aim of modification (or calibration) is to find γ_{L_n}, so that a constant level of reliability is maintained for any life-time of *n* years. Details of calculations are provided in **Appendix B** and the final solution is repeated here:

$$\gamma_{L_n} = 1.6\left[1 + \kappa \ln\left(\frac{n}{50}\right)\right] \tag{2.30}$$

Derived from Equation B13, κ or life-time modification coefficient depends on the element, limit state (failure mode) and as **Table 2.5** shows, to a lesser degree, the live load ratio, ρ, while it is independent of *n*.

Minimum live load factor

Similar to Equations 2.22 and 2.25, Equation 2.30 is not recommended for n<1 year. Live load factors belonging to shorter life-times are not of considerable practical interest; however, they are briefly discussed here for completeness.

The minimum value of the live load factor corresponds to the shortest of life-times, i.e., APT, therefore the limit for Equation 2.30 may be obtained by following the same procedure detailed in **Appendix B** with the difference that the statistical parameters of the arbitrary-point-in-time live load (**Table 2.2**) replace those calculated from Equations 2.22 and 25. Again this minimum load factor depends on the element, ultimate limit state and the live load ratio, ρ. For instance, if ρ=0.50 then $\gamma_{L_{APT}}$=0.697 for columns subject to compressive force and $\gamma_{L_{APT}}$=0.639 for slabs subject to flexure. With these two values calculated for the elements demanding the highest and lowest reliability indices respectively, **Fig. 2.3** depicts the variation of the live load factors for these elements, limit states and the live load ratio of 0.50 over time spans ranging from APT to 100 years.

Simplification for practical purposes

As **Fig. 2.3** reveals, the live load factor curves for different elements are virtually inseparable, bearing in mind that the curves for the live load factors of beams, under flexure or shear (not shown in the graph), lie between the two curves relative to columns and slabs. This calls for a simplification of Equation 2.30 which enhances its workability and streamlines the structural analysis:

$$\gamma_{L_n} = 1.6\left[1 + 0.09\ln\left(\frac{n}{50}\right)\right] \geq 0.65 \qquad (2.31)$$

Equation 2.31 is employable regardless of the live load ratio, element type or limit state. **Appendix C** recounts the details of this reformulation which, certainly, is not the sole approximation method and engineers, as they deem fit, can derive their own formulae.

2.7 EXAMPLE OF APPLICATION: REPAIR OF AN EXISTING BUILDING

A simple example illustrates how including the expected life-time of a building can influence and guide the process of the repair of a building.

An existing RC building must sustain a dead load of D=60 psf (L=3.0 kN/m^2) and a live load of L=80 psf (L=4.0 kN/m^2) which, according to ACI 318-11, leads to the required strength of the structural members of $U_{required}$ =(1.2)(60 psf)+(1.6)(80 psf)= 200 psf (10.0 kN/m^2). Due to the structural inadequacies detected in the building, it is determined that its structural members have an average capacity of $ØR_N$=184 psf (9.2 kN/m^2) which corresponds to the same dead load but a live load of only L=70 psf (L=3.5 kN/m^2). The design objective is to determine a) the required level of repair, if the remaining life of the structure after repair is required to be 10 years and b) the remaining life-time of the building, for the same level of safety of 50 years, if no repair is implemented.

a) Recalling Equation 2.31(for simplicity, although the more accurate form of Equation 2.30 is also possible) with n =10 years, the live load factor, γ_{L_n}, may be modified to 1.368. Hence $U_{required}$=(1.2)(60 psf)+(1.368)(80 psf)=181 psf (9.1 kN/m^2) < $U_{existing}$=184 psf (9.2 kN/m^2) which rejects the necessity of repair.

b) The expected life-time of an RC structure must satisfy Equations 2.29. For the building in the example this leads to γ_{L_n}=1.40. Reversing Equation 2.31 and solving for n, the remaining unrepaired life-time is approximately 12 years.

2.8 CONCLUSIONS

To adjust the live load factor of ACI 318-11, γ_L=1.6, based on the expected life-time of structures, the live load factor is derived as a function of the expected life-time, for common reinforced concrete elements and ultimate limit states (or failure modes). To that end, the statistical data available in the literature is used and expanded by describing the probabilistic parameters of live load as functions of life-time using the statistical model of extreme value distribution Type I. Assuming that the design requirements of ACI 318-11 are based on a life-time of 50 years, the live load factors, while different for different element types, converge to the factor stipulated by ACI 318-11 when the life-span approaches 50 years. In other words, all the factors are, as expected, ascending functions of life-time with a fixed value of 1.6 at n=50 years. However, due to their closeness and for the convenience of use in practice, all these curves are condensed in one to propose a unique and simple formulation for the variation of the live load factor with the expected life-time. This modified factor allows engineers to optimize their design without compromising the safety of structures.

The method detailed in this chapter is applicable to other time-dependent loads (e.g., seismic or wind loads) and other load combinations in order to generate load factors that are tied to the expected life-time.

The reliability analysis presented in this chapter involves some approximation as, in essence, it does not discriminate between normal and non-normal random variables. This can be alleviated by adopting numerical methods that incorporate the distribution of each random variable into calculations. The equations derived in this study may then be used to generate a semi-analytical solution for live load factors based on numerical outcomes which is the subject of the next chapter.

Table 2.1: Statistical parameters of resistance for cast-in-place RC members [1]

Structural type	**Limit state**	**Bias (λ_R)**	**CoV(δ_R)**	**Distribution**
Beam	Flexure[2]	1.190	0.089	Lognormal
Beam	shear	1.230	0.109	Lognormal
Slab	Flexure[2]	1.077	0.146	Lognormal
Tied column	compression	1.260	0.107	Lognormal

[1] Nowak and Szerszen (2003)

[2] Tension controlled

Table 2.2: Statistical parameters for load components

Load component	**Arbitrary point-in-time (APT)**			**50-year life-time**		
	Bias[1] (λ)	**CoV**[1] (δ)	**Distribution**[2]	**Bias**[1] (λ)	**CoV**[1] (δ)	**Distribution**[2]
Dead load[3]	1.05	0.10	Normal	1.05	0.10	Normal
Live load	0.24	0.65	Gamma	1.00	0.18	Type I

[1] Nowak and Szerszen (2003)
[2] Ellingwood and Galambos (1982)
[3] Cast-in-place

Table 2.3: Statistical parameters of live loads

Life-time (n years)	**Bias** (λ_{L_n})	**CoV** (δ_{L_n})
APT	0.240	0.650
1	0.452	0.398
5	0.677	0.266
10	0.774	0.233
25	0.903	0.199
50	1.000	0.180
100	1.097	0.164

Table 2.4:Target reliability indices, β_{50}, as a function of ρ for cast-in-place RC members

			$\boldsymbol{\beta_{50}}$			
Structural type	**Limit state**	**Ø**(2)	ρ=0.25	ρ=0.50	ρ=0.75	ρ=1.00
Beam	Flexure(1)	0.90	3.83	4.24	4.34	4.28
Beam	shear	0.75	4.39	4.69	4.80	4.81
Slab	Flexure(1)	0.90	2.12	2.45	2.64	2.75
Tied column	compression	0.65	5.21	5.47	5.57	5.57

(1)Tension controlled

*(2)*ACI 318-11

Table 2.5: Life-time modification coefficient, κ, as a function of ρ for cast-in-place RC members

		κ			
Structural type	**Limit state**	ρ=0.25	ρ=0.50	ρ=0.75	ρ=1.00
Beam	Flexure[(1)]	0.088	0.091	0.091	0.091
Beam	shear	0.085	0.088	0.087	0.086
Slab	Flexure[(1)]	0.096	0.100	0.102	0.103
Tied column	compression	0.081	0.083	0.082	0.081

[(1)]Tension controlled

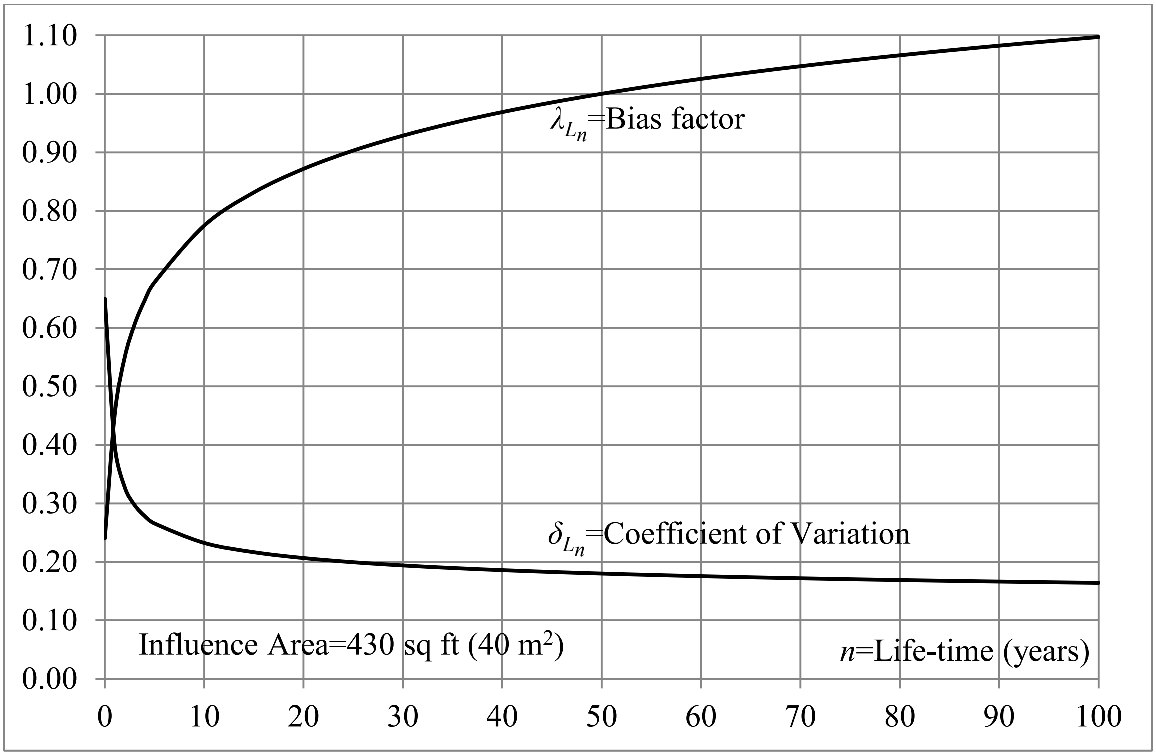

Figure 2.1:Statistical parameters of live load (λ_{L_n}, δ_{L_n}) vs. life-time (n)

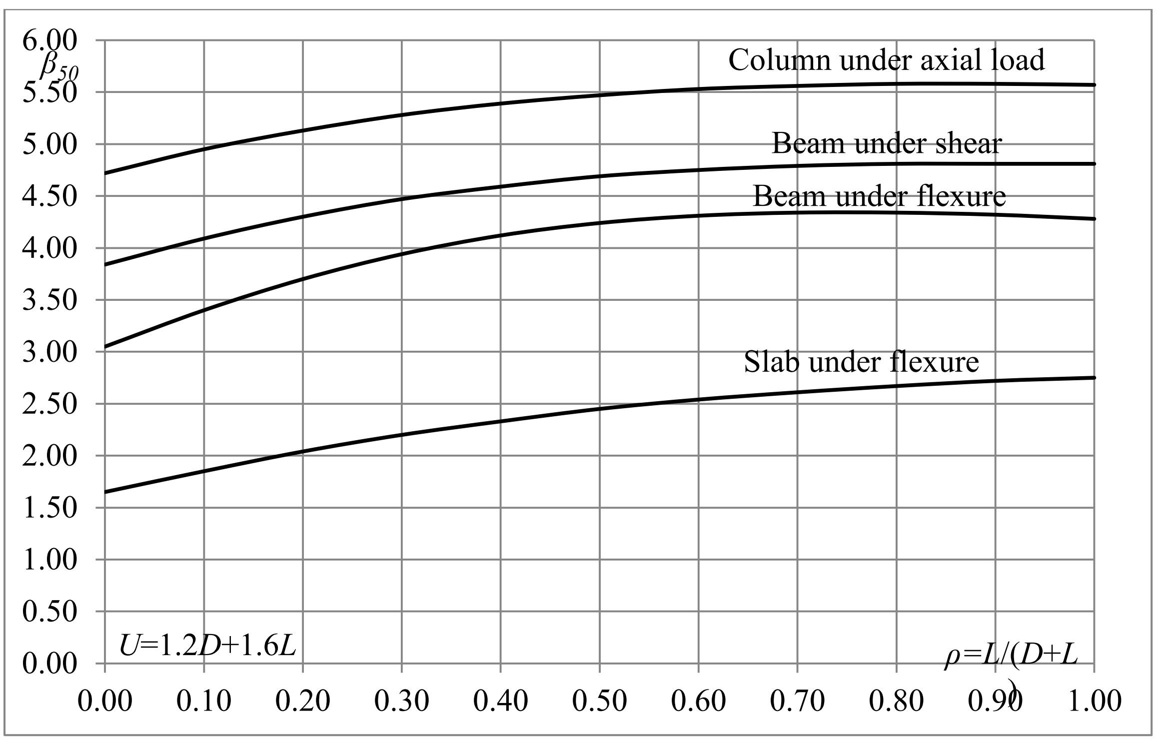

Figure 2.2: Target reliability indices for different cast-in-place RC elements

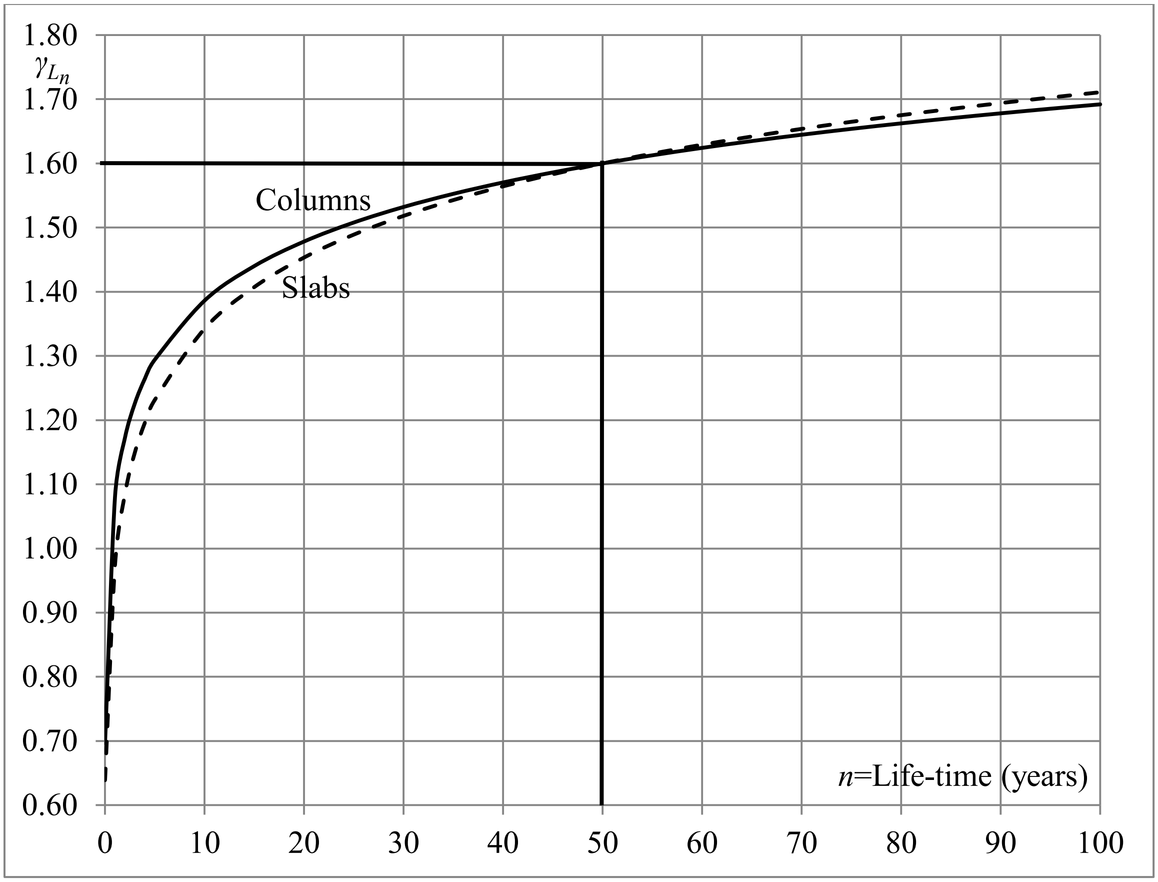

Figure 2.3: Live load factors for columns and slabs (ρ=0.50)

CHAPTER 3

3. STUDY II: NUMERICAL APPROACH TO LIVE LOAD FACTORS FOR RC STRUCTURES AS FUNCTIONS OF LIFE-TIME

3.1 BACKGROUND

Summary of study I

When conventional reinforced concrete (RC) structures are concerned the dominant design load combination against the gravitational loads, according to ASCE 7-10 and ACI 318-11, is:

$$U = 1.2D + 1.6L \tag{3.1}$$

Where *U* is the required strength and *D* and *L* are dead and live loads, or their related internal moments and forces. The live load is, naturally, time-dependent which binds Equation 3.1 to the intended life-time of the structure, for which a span of 50 years is the standard assumption. The aim is simply to generalize Equation 3.1 as a function of the expected life-time, *n*, or:

$$U_n = 1.2D + \gamma_{L_n} L \tag{3.2}$$

Where γ_{L_n} is a function of *n*, e.g., $\gamma_{L_{50}}$=1.60. As a preamble to this study, an approximate analytical solution was presented in Chapter 2 which yielded results in the general form of:

$$\gamma_{L_n} = \gamma_L \left[1 + \kappa \ln\left(\frac{n}{50}\right)\right] \tag{3.3}$$

Where, κ, the life-time modification coefficient is a function of two parameters: the limit state (failure mode) and $\rho = L/(D+L)$.

Objective

This chapter tries to overcome the deficiencies of the analytical solution presented in the previous chapter (Study I) using a numerical method (Rackwitz-Fiessler procedure) while benefiting from the approximate analytical solution to mold the numerical results into an analytical format. The two objectives of this study are: a) to use the numerical Rackwitz-Fiessler procedure to refine the approximate analytical results obtained in the previous study; and, b) to offer an extensive comparison of the mainstream reliability analysis methods (i.e., analytical, numerical and simulation techniques) which are applied in practical cases. The latter can be regarded as an investigation on the extent of efficiency and accuracy of each method.

3.2 OUTLINE

The contents of this chapter include a discussion of:

- The "target reliability index" is defined as the reliability index of a cast-in-place RC element complying perfectly with the load-resistance requirements of ACI 318-11, when the statistical parameters of the live load are derived for the duration of 50 years.
- The target reliability indices are calculated by the Rackwitz-Fiessler method and are juxtaposed with the results obtained by the conventional assumption that the limit state conforms to the normal distribution.
- The state of failure is simulated numerically via Monte Carlo method and the target reliability indices are recalculated. This stage provides proof positive of the enhancement in accuracy achieved by the Rackwitz-Fiessler method.

- For any life-time of n years, the statistical parameters of the live load are modified according to Chapter 2 and the live load factor is calibrated so that the target reliability is attained.
- The live load factor is formulated as a function of the life-time and conversely the expected life-time is described as a function of the live load capacity of an element and its design live load.

3.3 CALCULATION OF RELIABILITY INDEX: RACKWITZ-FIESSLER PROCEDURE

The first stage of the process is the calculation of the target reliability index, β_T, which, in this study, is redefined as, β_{50}, the reliability index of an element if the strength reduction (Ø) and load factors (γ) are based on ACI 318 and the statistical parameters of the live load are based on a life-time of 50 years. Study I provides these values of β_{50} based on the simplifying assumption that load and resistance are random variables that follow the normal distribution (top part of **Table 3.1**).

To achieve higher accuracy, the numerical Rackwitz-Fiessler procedure can be utilized which takes into account the probabilistic distribution of the random variables. This procedure is detailed and developed in **Appendix D** to accommodate the limit state investigated in this study. The middle part of **Table 3.1** shows the target reliability indices for a few selected values of $\rho=L/(D+L)$ calculated by this procedure. As an example, **Table 3.2** details how $\beta_T=\beta_{50}$=3.87 is calculated for an RC beam subject to flexural failure (ultimate limit state) caused by equal nominal live and dead loads (ρ=0.50). It should be noted that the first cycle of the Rackwitz-Fiessler is equivalent to the assumption that all the variables are normally distributed, as in Equation A17 or equally in Study I, and leads to the same results. For the instance of this case, β=4.24 from both the normal assumption part of **Table 3.1** and the first cycle in **Table 3.2**. The subsequent values of β in **Table 3.2** show the transition towards the eventual convergence, β=3.87, which is the corresponding value of β_{50} obtained by the Rackwitz-Fiessler procedure (the middle part of **Table 3.1**). For the cases investigated in **Table 3.1,** the relative difference between the results of the two methods reaches a maximum of 29% and averages at 12%.

Fig. 3.1 portrays a comparison between the two approaches in the case of beams. Significant differences, especially for cases where the live load accounts for the majority of the total applied load, necessitate that the error associated with the two approaches be estimated. In this study the Monte Carlo simulation, discussed in the following section, is employed to arbitrate between the former two methods and provide a sense of accuracy of each.

3.4 CALCULATION OF RELIABILITY INDEX: VALIDATION BY MONTE CARLO SIMULATION

Monte Carlo method is simply a brute force technique that generates samples of numerical data (e.g., load and resistance) and then observes the number of times an event of concern (e.g., failure) occurs. **Appendix E** provides the details of the method by which any level of precision is theoretically obtainable; however, it also points out its computational intensity if a high reliability index is to be calculated with a high level of accuracy, the reason that renders the two aforesaid methods (Rackwitz-Fiessler and normal assumption) , even with their recognized limitations, of great practical interest.

The bottom part of **Table 3.1** shows the results of the Monte Carlo simulations which are in close accordance with the Rackwitz-Fiessler method (less than 1% difference on average with a maximum of 2.5 %). With the target reliability indices validated, the actual objective of this study and its contribution to the reliability literature is explained in the following sections.

3.5 CALIBRATION OF LIVE LOAD FACTOR BASED ON THE EXPECTED LIFE-TIME

When the target reliability is calculated by the Rackwitz-Fiessler procedure (middle part of **Table 3.1**), the parameters λ_L, δ_L and γ are replaced by their counterparts that correspond to a life-time of *n* years, λ_{L_n}, δ_{L_n} and γ_n as formulated in Study I. For each life span, the load factor γ_n is adjusted so that this target reliability is attained. This requires a trial and error solution for γ_n, with each trial going through the iterative procedure detailed in **Appendix D**. The modified live load factor, γ_{L_n}, is then back calculated from Equation B2. **Table 3.3** shows the calibrated live load factors (γ_{L_n}) for ρ=0.50.

Example of application

A simple example may explain **Table 3.3** more clearly. An RC beam is subject to equal dead and live loads of 50 psf (2.5 kN/m^2) which also produce equal internal moments. The aim is to calculate its required flexural strength, U_n, if

a) the beam is to be designed according to ACI 318-11.

b) the beam is being examined for possible repair as its existing factored capacity is evaluated as $ØR_N$ = 130 psf (6.5 kN/m^2), while it is required to function for only 5 more years.

c) the beam is part of an important building with an expected life-time of 100 years.

Since D=L then ρ= L/ (D+L)= 0.50 and **Table 3.3** is applicable. Therefore:

a) U_{50}=140 psf (7.0 kN/m^2) from either Equation 3.1 or equally by substituting $\gamma_{L_{50}}$=1.600 (from **Table 3.3)** into Equation 3.2.

b) From **Table 3.3**, γ_{L_5}=1.314, and by substituting into Equation 3.2: U_5=125.7 psf (6.3 kN/m^2)< $\text{Ø}R_N$ =130 psf (6.5 kN/m^2), hence, no repair is needed although the beam is deficient according to ACI 318-11 which demands a strength of U_{50}=140 psf (7.0 kN/m^2) from part (a).

c) From **Table 3.3**, $\gamma_{L_{100}}$=1.686, and by substituting into Equation 3.2: U_{100}=144.3 psf (7.2 kN/m^2). A slight addition to the strength, 4.3 psf (0.2kN/m^2), drastically lengthens the life-time.

For other loading proportions (different values of ρ) tables similar to **Table 3.3** can be developed which are left out for brevity; instead, a generalized solution that practically covers all the cases is formulated in the next section.

3.6 UNIFIED FORMULATION OF LIVE LOAD FACTORS

To obtain a formula for the live load factors from the numerical values a regression curve of must be fitted to each column of **Table 3.3**. Finding the optimal curve type to be fitted to a set of numerical data is normally a matter of trial and error. In this study, however, the analytical although approximate solution (Chapter 2) provides the benefit of having a natural candidate that with minor modifications fits extremely well into the numerical results. Furthermore, such a regression curve has the additional advantage of forecasting the results beyond the original data points with adequate certainty, while the extrapolated tails of the curves selected by trial and error must always be treated with extreme caution.

Equation 3.3 is, therefore, fitted to the columns of **Table 3.3,** where the unknown κ or life-time modification coefficient, is calculated from conventional regression procedures. **Table 3.4** shows these results and others that pertain to a few selected values of ρ. Compared to the analytical results (Chapter 2) which demonstrated little and slightly erratic sensitivity to ρ, the numerical results follow a general and noticeable declining trend as ρ, the share of live load from the total load, increases. The gap between the two methods also widens with the increase of live load or ρ.

Fig. 3.2 and **Fig. 3.3** compare the results obtained in this study with those from Study I in the cases of a beam subject to flexure and a column subject to compressive axial load. For both cases D=L or ρ=0.50. Equation 3.3, however, is singular if n=0 which puts the start point of the curves out of the range of the validity of Equation 3.3.The next section deals with this point.

3.7 MINIMUM VALUE OF LIVE LOAD FACTORS

A restriction must be imposed on the minimum acceptable value of n in Equation 3.3. Logically the minimum modification factor corresponds to that of the shortest life span or APT. Therefore values of n that yield live load factors smaller than those corresponding to the arbitrary–point-in-time (**Table 3.5**) are to be neglected, or:

$$\gamma_{L_n} = \gamma_L \left[1 + \kappa \ln\left(\frac{n}{50}\right)\right] \geq \gamma_{L_{APT}} \tag{3.4}$$

$\gamma_{L_{APT}}$ is obtained in the same manner of γ_{L_n}, taking into account that in the APT case the probabilistic distribution of the live load conforms to the gamma distribution instead of EVD type 1, as discussed in more detail in **Appendix D**. Conservatively, however, the inequality may be replaced by limiting the life-time to a minimum of 1 year (n≥1).

3.8 APPLICATION TO CALCULATION OF EXPECTED LIFE-TIME AS A FUNCTION OF LOAD AND RESISTANCE

The obvious application of mathematical procedure developed in this chapter is to adjust the required strength according to the expected life-time of a new structure or the remaining life-time of an existing one. This was explained earlier by a simple example, but this formulation, if reversed, can also be an aid to engineers attempting to estimate the expected life of a structure based on its design capacity and design loads.

If an RC element is optimally designed then, according to ACI 318, its nominal strength, R_N, must uphold Equation 3.5:

$$\emptyset R_N = 1.2D + 1.6L \tag{3.5}$$

Where $\emptyset$ is the strength reduction factor. While the nominal strength of an over or under-designed element satisfies Equation 3.6:

$$\emptyset R_N = 1.2D + 1.6L' \tag{3.6}$$

Where L' is the nominal live load capacity of the member and accounts for the deviation from the loading requirements. If Equations 3.2 and 3.6 are equated ($Un = \emptyset R_N$) the life-time of the element, n, or the period during which its capacity satisfies the strength requirement can be derived as:

$$n = 50e^{\left(\frac{L'}{L}-1\right)/\kappa} \tag{3.7}$$

Again, L' is the nominal live load that a member can sustain, L is the design live load stated by the code and κ, is selected from **Table 3.4**. A simplified example can show how life-time is affected by a change in use of a building.

A building, based on its function, must be capable of carrying a live load of L=50 psf (2.5 kN/m^2). It is also assumed that the dead load has an equal magnitude or D=50 psf (2.5

kN/m^2) which leads to U=140 psf (7.0 kN/m^2). However, the building is slightly over-designed and its structural members have an average capacity of $ØR_N$=150 psf (7.5 kN/m^2) which according to Equation 3.6 can be divided into the unchanged dead load of D=50 psf (2.5 kN/m^2) and the new live load of L'=56.25 psf (2.8 kN/m^2). The building is now being converted to a different use which, according to the load requirements, necessitates a design live load of L=60 psf (3.0 kN/m^2). The effect of this conversion is to be investigated on the expected life-time of each element type.

Since, dead and live load are approximately equal before and after conversion, $\rho \approx 0.5$, and with a slight approximation, corresponding values of κ from **Table 3.4** may be substituted in Equation 3.7. **Table 3.6** shows how the initial over-strength and subsequent under-strength greatly influence the expected life-time of the building and the probability of failure of each member type. Subtly, it also reveals that degree of sensitivity of the elements to over or under-strength conforms to the hierarchy of their importance or reliability: The most sensitive are the columns, the least are the slabs while beams are in between. Again for beams, shear limit state is more decisive in defining the life-time as compared to the more ductile and less sudden flexural limit state.

The procedure described in this study can be applied on other time dependent loads, snow, wind and earthquake or other structural elements to describe their load factors as functions of life-time for the limit states considered.

3.9 CONCLUSIONS

The following results are drawn from this study:

- Live load factors as functions of the expected life-time are derived for common reinforced concrete elements and ultimate limit states (or failure modes), using the statistical data available in the literature and expanding the probabilistic parameters of live load as a function of life-time by means of the statistical model of extreme value distribution Type I. The numerical Rackwitz-Fiessler procedure of reliability analysis is applied to this data, while a simplified analytical method (Study I) provides the basis upon which the numerical results are shaped into a uniform and simple analytical format (Equation 3.3). Assuming that the design requirements of ACI 318-11 aiming at a life-time of 50 years, the live load factors are, as expected, ascending functions of the life-time, n, which converge at the fixed value of γ_L=1.60 at n=50 years, but differ in the life-time modification coefficient, κ, which varies according to the element type, limit state and the ratio of live load to the total load.

- By providing a method to calculate the influence of any under-strength on the life-time or equally the probability of failure during a certain period, live load modification equations may also be interpreted as a means to appraise the necessity of repair for a structure. Expected life of a member as portrayed by Equation 3.7 is an exponential function of the live load demand (L), its live load capacity (L') and its failure mode (κ). It is demonstrated how even a slight deficiency in strength may have dramatic implications on the probability of failure and make rehabilitation required. Conversely, a seemingly insignificant over-strength may drastically prolong the expected life of a structure. The latter implies that for important structures which are expected or needed to last considerably more than 50 years, comparatively small modifications in design can provide both safety and economy.

- A byproduct of this study is a relatively comprehensive comparison of the common methods utilized in the reliability analysis: the simple assumption that the limit state conforms to a normal distribution, the more sophisticated Rackwitz-Fiessler method which observes, to some extent, the effect of different statistical distribution and finally the Monte Carlo method of simulation by which any degree of accuracy is theoretically attainable, so long as the computational obstacles can be overcome. For the cases investigated, as the portion of the live load with respect to the total load grows, the first method leans towards over-predicting the reliability index. Thus, it is recommendable that whenever the live load forms the majority of the total load, either of the two latter methods, Rackwitz-Fiessler or Monte Carlo, be employed to perform the reliability analysis.

Table 3.1: Target reliability indices, β_{50}, as a function of ρ for cast-in-place RC members

Method	Structural type	Limit state	β_{50} ρ=0.25	ρ=0.50	ρ=0.75	ρ=1.00
Normal distribution	Beam	Flexure[(1)]	3.83	4.24	4.34	4.28
	Beam	shear	4.39	4.69	4.80	4.81
	Slab	flexure[(1)]	2.12	2.45	2.64	2.75
	Tied column	compression	5.21	5.47	5.57	5.57
Rackwitz-Fiessler procedure	Beam	flexure[(1)]	4.15	3.87	3.55	3.33
	Beam	shear	5.22	4.74	4.31	4.02
	Slab	flexure[(1)]	2.40	2.72	2.73	2.67
	Tied column	compression	6.39	5.59	5.02	4.64
Monte Carlo simulation	Beam	flexure[(1)]	4.09	3.85	3.53	3.32
	Beam	shear	5.16	4.72	4.31	4.02
	Slab	flexure[(1)]	2.34	2.67	2.72	2.67
	Tied column	compression	6.35	5.57	5.02	4.65

[(1)]Tension controlled

Table 3.2: Calculation of β_{50} for RC beams in flexure (ρ=0.50)[1]

	Iteration cycle				
Parameter	1	2	3	4	5
σ_d	0.047	0.047	0.047	0.047	0.047
μ_d	0.472	0.472	0.472	0.472	0.472
σ_l^e	0.081	0.136	0.200	0.223	0.227
μ_l^e	0.450	0.377	0.228	0.160	0.148
σ_q^e	0.094	0.144	0.206	0.228	0.232
μ_q^e	0.922	0.849	0.700	0.633	0.620
σ_r^e	0.148	0.100	0.118	0.124	0.125
μ_r^e	1.666	1.566	1.625	1.637	1.639
d^*	0.527	0.524	0.509	0.506	0.505
l^*	0.608	0.806	0.887	0.902	0.904
r^*	1.135	1.331	1.396	1.408	1.409
β	4.238[2]	4.072	3.887	3.873	3.873[3]

(1) For definitions of symbols see **NOTATIONS**.

(2) Similar to normal assumption: $\beta_T = \beta_{50} \approx 4.24$

(3) Convergence achieved: $\beta_T = \beta_{50} \approx 3.87$

Table 3.3: Live load factors, γ_{L_n} (ρ=0.50)

Life-time (n years)	**Structural type and limit state** RC beam cast-in-place, flexure[(1)]	RC beam cast-in-place, shear	RC slab cast-in-place, flexure	RC column cast-in-place, tied[(2)]
APT	0.802	0.826	0.709	0.840
1	1.115	1.164	0.991	1.218
5	1.314	1.342	1.238	1.374
10	1.402	1.421	1.347	1.443
25	1.512	1.520	1.490	1.530
50	1.600	1.600	1.600	1.600
100	1.686	1.678	1.710	1.667

[(1)]Tension controlled

[(2)]Axial compression

Table 3.4: Life-time modification coefficient, κ, as a function of ρ

		κ			
Structural type	**Limit state**	ρ=0.25	ρ=0.50	ρ=0.75	ρ=1.00
Beam	flexure[(1)]	0.086	0.078	0.074	0.073
Beam	shear	0.081	0.070	0.065	0.063
Slab	flexure[(1)]	0.100	0.098	0.093	0.089
Tied column	compression	0.073	0.061	0.057	0.055

[(1)]Tension controlled

Table 3.5: Minimum live load factors, $\gamma_{L_{APT}}$

		$\gamma_{L_{APT}}$			
Structural type	**Limit state**	ρ=0.25	ρ=0.50	ρ=0.75	ρ=1.00
Beam	flexure[(1)]	0.800	0.802	0.842	0.870
Beam	shear	0.810	0.826	0.886	0.928
Slab	flexure[(1)]	0.720	0.709	0.744	0.774
Tied column	compression	0.800	0.840	0.925	0.974

[(1)]Tension controlled

Table 3.6: Effect on life-time of a hypothetical RC building

		Life-time(years)	
Structural type	**Limit state**	**Before conversion** L=50 psf (2.5 kN/m^2)	**After conversion** L=60 psf (3.0 kN/m^2)
Beam	flexure[(1)]	248	22
Beam	shear	298	20
Slab	flexure[(1)]	179	25
Tied column	compression	388	18

[(1)]Tension controlled

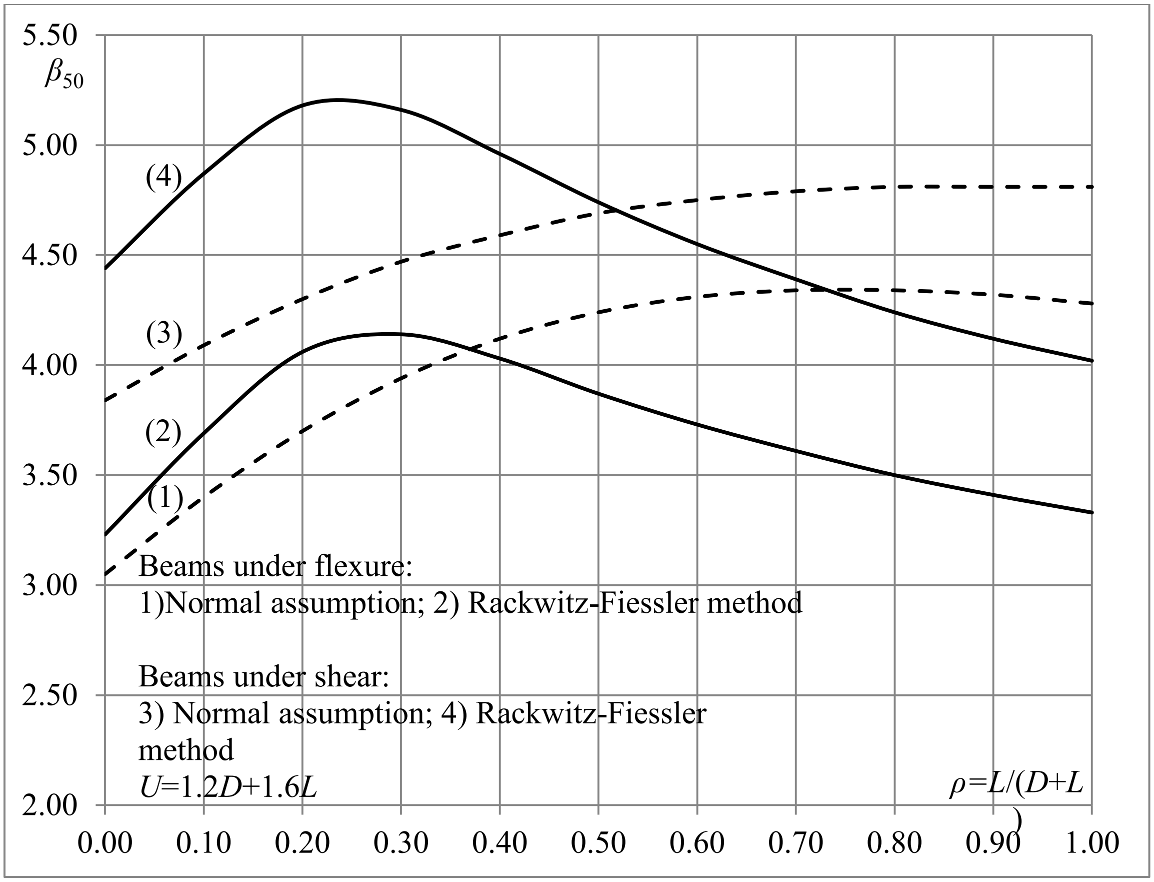

Figure 3.1: Target reliability indices, β_{50}, as a function of ρ for cast-in-place RC beams (Rackwitz-Fiessler method compared to normal distribution assumption)

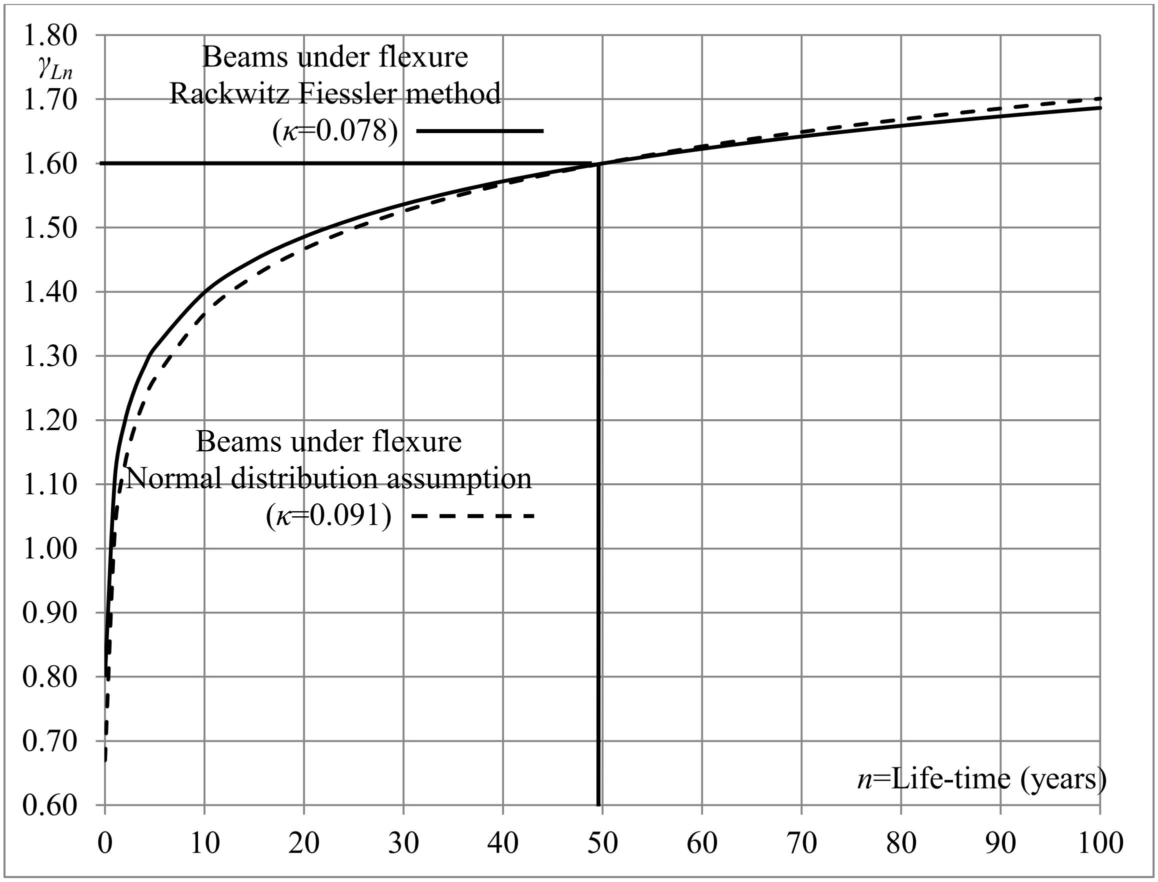

Figure 3.2: Live load factors for RC beams subject to flexure (ρ=0.50)

Analytical vs. numerical results

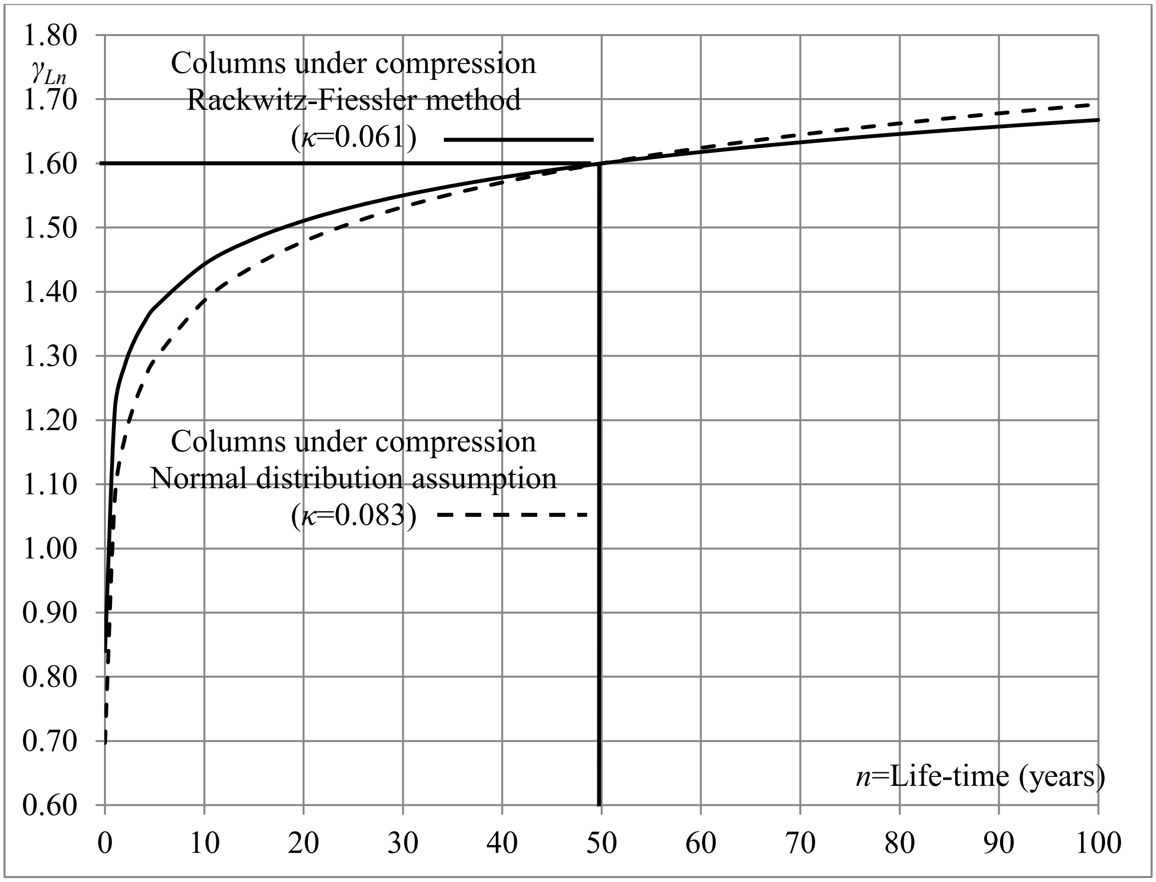

Figure 3.3: Live load factors for RC columns subject to axial compression (ρ=0.50)

Analytical vs. numerical results

CHAPTER 4

4. STUDY III: RELIABILITY ANALYSIS OF CONCRETE BEAMS INTERNALLY REINFORCED WITH FRP BARS

4.1 BACKGROUND

The ever-increasing use of FRP materials in construction, both in new and existing structures, calls for a deeper investigation into the selection of the strength reduction (or variously called safety) factors imposed on their design equations, as the current values have been typically chosen based on judgment and consensus and are still in need of hard evidence for validation. This validation can be attained by the reliability analysis that links the probability of failure to the load and safety factors, providing a basis for their calibration to achieve desired levels of safety. Conventionally, the reliability index is defined as an indicator of the probability of failure of a member with a resistance of R, against the loads it may experience during its life-time, Q; both Q and R being random variables. This, however, poses a few obvious difficulties due to presence of load parameters in the calculations:

- Compared to resistance, statistical parameters of loading are far more difficult to obtain, due to the vast number of factors affecting load.
- Load and resistance, being of different nature, follow different statistical distributions which, especially in the case of multiple load cases, makes the problem of calculating the reliability index less tractable.

• Reliability analysis has to be performed for several types of loads and load combinations.

• For each load combination, covering the whole range of plausible loadings makes the calculations cumbersome, especially when more than two loads are involved.

The idea central to this study is to calibrate safety factors of the elements reinforced with new materials not by setting them against loads, but by comparing them to elements of the same capacity, but made of better-established and better-known materials. In other words, if the current safety factors for steel-reinforced concrete and its associated load factors are taken for granted, as there is little doubt about its performance when designed according to code, how should the safety factors of FRP-reinforced members be proportioned so that the same level of safety is attained? This study elaborates this concept, applies it to the case of beams internally reinforced with FRP bars and proposes revised strength reduction (safety) factors for use in FRP design guidelines.

4.2 OUTLINE

The contents of this chapter, in sequential order, may be summarized as:

- By employing the concept of reliability index, an interim index of "comparative reliability" is proposed that bypasses the loading variables and weighs the resistances of two structural elements with the same ultimate limit state directly against each other.
- The comparative reliability index is then related to the conventional target reliability to allow a simple calculation method of the strength reduction factor for elements whose strength reduction factor is yet to be calibrated. This is achieved

by comparing a pair of elements which experience the same failure mode and yet only one holds a validated level of safety (i.e., strength reduction factor and reliability index).

- This concept is put into practice by calculating flexural and shear strength reduction factors for FRP reinforced concrete members by comparison with conventional steel-reinforced concrete beams possessing the same ultimate capacity.
- As a result, a revised set of strength reduction factors and deign provisions is proposed for use in FRP design guidelines for shear and flexure.

By following these steps, this study introduces a method to extrapolate the strength reduction factors of new construction materials from the experience gained with conventional ones. Such a method, without compromising safety, prevents penalizing new materials. Accordingly, this study attempts to develop consistency between the flexural and shear strength reduction factors of ACI 318 and ACI 440 documents.

4.3 COMPARATIVE RELIABILITY

Let R_1 and R_2 be the resistance of two elements 1 and 2 with the same ultimate limit state. The comparative reliability index, β_c, is defined as a measure of the probability of element 1 possessing a lower level of resistance than element 2. Calculation of the comparative reliability index of two elements is almost identical to calculation the of the reliability index, β, for lognormal load and resistance variables found in Nowak and Collins (2000) and Haldar and Mahadevan (2000); nevertheless, the procedure is detailed here because of the modification due to the removal of load from the equation.

The means of the random resistance variables R_1 and R_2 are denoted by μ_1 and μ_2 while σ_1 and σ_2, respectively, denote their standard deviations. Since resistance can only take positive values, it is believed to conform to lognormal distribution (Ellingwood and Galambos 1982) and so do R_1 and R_2. Therefore, another lognormal random variable G can be introduced as:

$$G = \frac{R_2}{R_1} \tag{4.1}$$

$$\ln(G) = g = \ln(R_2) - \ln(R_1) \tag{4.2}$$

The probability of the event $R_2 < R_1$ or $p_c = P(R_2 < R_1)$ can be written as:

$$p_c = P(G < 1) = P(g < 0) \tag{4.3}$$

$\ln(R_1)$, $\ln(R_2)$ and subsequently g are normal variables, therefore:

$$P(g < g_0) = \Phi(\frac{g_0 - \mu_g}{\sigma_g}) \tag{4.4}$$

Where g_0 is an arbitrary value, Φ is the cumulative distribution function (CDF) of the standard normal distribution, and μ_g and σ_g are, respectively, the mean and standard deviation of g. Hence, by substituting g_0=0 in Equation 4.4:

$$p_c = \Phi\left(-\frac{\mu_g}{\sigma_g}\right) = \Phi\left(-\frac{1}{\delta_g}\right) \tag{4.5}$$

δ_g is the coefficient of variation of g. Defining Φ^{-1} as the inverse function of Φ, Equation 4.5 can be rewritten as:

$$\frac{\mu_g}{\sigma_g} = -\Phi^{-1}(p_c) = \Phi^{-1}(1 - p_c) = \beta_c \tag{4.6}$$

β_c is the "comparative reliability index" and can be calculated in terms of the statistical parameters of R_1 and R_2. The relationship between the statistical parameters of a lognormal variable, X, and its normal twin, $\ln(X)$, is (Nowak and Collins 2000):

$$\sigma^2_{\ln(X)} = \ln(1 + \delta^2_X) \tag{4.7}$$

$$\mu_{\ln(X)} = \ln(\mu_X) - \frac{\sigma^2_{\ln(X)}}{2} = \ln(\mu_X) - \frac{\ln(1+\delta_X^2)}{2} \tag{4.8}$$

As R_1 and R_2 are reasonably assumed to be statistically independent, from Equation 4.2 it follows that**:**

$$\mu_g = \mu_{\ln(R_2)} - \mu_{\ln(R_1)} \tag{4.9}$$

$$\sigma_g^2 = \sigma^2_{\ln(R_1)} + \sigma^2_{\ln(R_2)} \tag{4.10}$$

Replacing the statistical parameters of normal variables $\ln(R_1)$ and $\ln(R_2)$ in Equations 4.9 and 10 with those written in terms of R_1 and R_2 using Equations 4.7 and 4.8, one obtains :

$$\mu_g = \ln\left[\left(\frac{\mu_2}{\mu_1}\right)\sqrt{\frac{1+\delta_1^2}{1+\delta_2^2}}\right] \tag{4.11}$$

$$\sigma_g^2 = \ln(1+\delta_1^2) + \ln(1+\delta_2^2) \tag{4.12}$$

Where $\delta_1 = \delta_{R_1}$ and $\delta_2 = \delta_{R_2}$. Eventually, substituting Equations 4.11 and 4.12 into Equation 4.6, β_c, can be formulated as:

$$\beta_c = \frac{\mu_{\ln(R_2)} - \mu_{\ln(R_1)}}{\sqrt{\sigma^2_{\ln(R_1)} + \sigma^2_{\ln(R_2)}}} = \frac{\ln\left[\left(\frac{\mu_2}{\mu_1}\right)\sqrt{\frac{1+\delta_1^2}{1+\delta_2^2}}\right]}{\sqrt{\ln(1+\delta_1^2)+\ln(1+\delta_2^2)}} \tag{4.13}$$

4.4 RELATIONSHIP BETWEEN TARGET AND COMPARATIVE RELIABILITY INDICES

The comparative reliability index, β_c, is merely an intermediary parameter, as it does not reveal anything about the probability of failure which is measured by the reliability index β and its acceptable level, β_T, or target reliability.

Next is to relate the comparative reliability index, β_c, to the target reliability, β_T. The conventional reliability index, β, for lognormal load (Q) and resistance variables (R_1 or R_2) is defined as:

$$\beta(Q, R_1) = \frac{\mu_{\ln(R_1)} - \mu_{\ln(Q)}}{\sqrt{\sigma^2_{\ln(R_1)} + \sigma^2_{\ln(Q)}}} = \beta_T \tag{4.14}$$

$$\beta(Q, R_2) = \frac{\mu_{\ln(R_2)} - \mu_{\ln(Q)}}{\sqrt{\sigma^2_{\ln(R_2)} + \sigma^2_{\ln(Q)}}} = \beta_T \tag{4.15}$$

In Equations 4.14 and 4.15, it is assumed that the target reliability, β_T, or the expected level of safety is equal for both elements. To eliminate the square root and separate R_1 or R_2 from Q in Equations 4.14 and 4.15 an intermediary parameter, ε, is introduced (Haldar and Mahadevan 2000):

$$\varepsilon_1 = \frac{\sqrt{\sigma^2_{\ln(R_1)} + \sigma^2_{\ln(Q)}}}{\sigma_{\ln(R_1)} + \sigma_{\ln(Q)}} \tag{4.16}$$

$$\varepsilon_2 = \frac{\sqrt{\sigma^2_{\ln(R_2)} + \sigma^2_{\ln(Q)}}}{\sigma_{\ln(R_2)} + \sigma_{\ln(Q)}} \tag{4.17}$$

Similarly, to separate R_1 and R_2 in Equation 4.13:

$$\varepsilon_c = \frac{\sqrt{\sigma^2_{\ln(R_1)} + \sigma^2_{\ln(R_2)}}}{\sigma_{\ln(R_1)} + \sigma_{\ln(R_2)}} \tag{4.18}$$

ε_1, ε_2 and ε_c vary in a narrow range ($\sqrt{2}/2 \le \varepsilon \le 1.00$); hence, if their values are comparable which is a reasonable assumption, the following approximation can be made:

$$\varepsilon_1 \approx \varepsilon_2 \approx \varepsilon_c \tag{4.19}$$

Combining Equations 4.13 to 4.19, β_c can be rewritten as:

$$\beta_c \approx \frac{\sigma_{\ln(R_2)} - \sigma_{\ln(R_1)}}{\sigma_{\ln(R_1)} + \sigma_{\ln(R_2)}} \beta_T = \frac{\sqrt{\ln(1+\delta_2^2)} - \sqrt{\ln(1+\delta_1^2)}}{\sqrt{\ln(1+\delta_1^2)} + \sqrt{\ln(1+\delta_2^2)}} \beta_T \tag{4.20}$$

And from Equations 4.13 and 4.20:

$$\beta_c = \frac{\ln\left[\left(\frac{\mu_2}{\mu_1}\right)\sqrt{\frac{1+\delta_1^2}{1+\delta_2^2}}\right]}{\sqrt{\ln(1+\delta_1^2)+\ln(1+\delta_2^2)}} = \frac{\sqrt{\ln(1+\delta_2^2)}-\sqrt{\ln(1+\delta_1^2)}}{\sqrt{\ln(1+\delta_1^2)}+\sqrt{\ln(1+\delta_2^2)}}\beta_T \tag{4.21}$$

Equation 4.21 can be further simplified by taking advantage of Taylor expansion:

$$\ln(1+x) \approx x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots \qquad \text{if } |x| < 1 \tag{4.22}$$

Assuming that $\delta_X \leq 0.30$, within a margin of error of about 2%, Equation 4.7 may be simplified as:

$$\sigma_{\ln(x)} = \sqrt{\ln(1+\delta_X^2)} \approx \delta_X \tag{4.23}$$

Hence, if both δ_1 and $\delta_2 \leq 0.30$, which is a common situation, Equations 4.13 and 4.21 can be simplified as:

$$\beta_c = \frac{\ln\left[\left(\frac{\mu_2}{\mu_1}\right)\right]}{\sqrt{\delta_1^2+\delta_2^2}} = \frac{\delta_2-\delta_1}{\delta_1+\delta_2}\beta_T \tag{4.24}$$

β_c has concluded its part by assisting to establish a load free relationship between β_T and the statistical parameters of R_1 and R_2, as formulated in Equations 4.21 or 4.24, which are identified as the comparative reliability equations. The target reliability, β_T, has its preset minimum values based on consequence of failure and incremental cost of safety as shown in **Table 4.1** for reinforced concrete members made of ordinary concrete (Nowak and Szerszen 2003). **Table 4.1** also presents the safety factors stipulated by ACI 318-11, coefficients of variations (CoV or δ) and bias factors (λ). The latter parameter is defined in the following section.

4.5 CALIBRATION OF STRENGTH REDUCTION FACTORS

When rewritten as a function of the strength reduction factors, Ø, Equation 4.24 (or 4.21) can be used calibrate to those factors. Structural elements 1 and 2 are comparable only if they are equal in their ultimate design capacity or:

$$\phi_1 N_1 = \phi_2 N_2 \tag{4.25}$$

Where $Ø_i$ indicates the strength reduction factor and N_i is the nominal strength (design strength) of each element (i=1, 2). Element 1(e.g., steel RC beam) serves as the benchmark for calibration of the safety factor of element 2 (e.g., FRP RC beam). In other words, $Ø_1$ and the desired level of reliability, β_T, are assumed to be known and $Ø_2$ has to be calculated from them.

The bias factor, λ_i, is defined as the ratio of the mean value, μ_i, to the nominal value, N_i, of a random variable or in this case R_1 and R_2, therefore:

$$\lambda_1 = \frac{\mu_1}{N_1} \tag{4.26}$$

$$\lambda_2 = \frac{\mu_2}{N_2} \tag{4.27}$$

It can be concluded from Equations 4.25 to 4.27 that:

$$\frac{\mu_2}{\mu_1} = \frac{\phi_1}{\phi_2}\frac{\lambda_2}{\lambda_1} \tag{4.28}$$

If δ_1 and $\delta_2 \leq 0.30$, substituting Equation 4.28 into Equation 4.24 results in:

$$\frac{\ln\left[\left(\frac{\phi_1\lambda_2}{\phi_2\lambda_1}\right)\right]}{\sqrt{\delta_1^2+\delta_2^2}} = \frac{\delta_2-\delta_1}{\delta_1+\delta_2}\beta_T \tag{4.29}$$

If δ_1 or $\delta_2 > 0.30$, then the simplification of Equation 4.21 is not suitable and Equation 4.30 provides better accuracy than Equation 4.29:

$$\frac{\ln\left[\left(\frac{\phi_1\lambda_2}{\phi_2\lambda_1}\right)\sqrt{\frac{1+\delta_1^2}{1+\delta_2^2}}\right]}{\sqrt{\ln(1+\delta_1^2)+\ln(1+\delta_2^2)}} = \frac{\sqrt{\ln(1+\delta_2^2)}-\sqrt{\ln(1+\delta_1^2)}}{\sqrt{\ln(1+\delta_1^2)}+\sqrt{\ln(1+\delta_2^2)}}\beta_T \tag{4.30}$$

In any event, $Ø_2$ can be calculated directly from Equations 4.29 or 4.30.

This concludes the first part of this chapter that deals with the introduction and formulation of the comparative reliability. The following part takes advantage of this concept to calculate the safety factors of concrete beams reinforced with FRP bars subject to flexure and shear.

4.6 FLEXURE-CONTROLLED FAILURE OF FRP RC MEMBERS

A reliability analysis for FRP-reinforced beams in flexure using the load combination of U=1.2D+1.6L for live to dead load ratios between 1 and 3, indicated reliability indices between 3.5 and 4.0 when Ø was set to 0.65 for both concrete crushing failure and FRP reinforcing bar rupture failure (Gulbrandsen 2005). Based on these results, ACI 440.1R-08, recommends a strength reduction factor of 0.55, if failure is due to FRP rupture, and 0.65 if failure is due to concrete crushing, with a linear variation in the transitional range between the two failure modes. **Table 4.2** summarizes the statistical data (bias factor and coefficient of variation) obtained from tests discussed in detail in Gulbrandsen (2005), its assumptions (minimum target reliabilities) and strength reduction factors adopted by ACI 440.1R-08.

Here the aim is to calibrate the FRP-RC strength reduction factors, not by using conventional reliability analysis as in Gulbrandsen (2005), but by taking advantage of the concept of comparative reliability. Using Equation 4.29, the strength reduction factors can be calibrated proportional to those of a steel RC beam whose statistical parameters are given in **Table 4.1**(Nowak and Szerszen 2003). In other words, λ_1 and δ_1 in Equation 4.29 correspond to the first row of **Table 4.1**, while λ_2 and δ_2, depending on the failure mode, are taken from the two rows of **Table 4.2** respectively. As a result, **Table 4.3** displays the calculated values of the safety factor, Ø, for different assumptions of the target reliability; however, only the first column (β_T=3.5) corresponding to the assumed safety target of ACI 318-11 is of primary interest while the remaining columns are merely presented to show the relatively low sensitivity of Equation 4.29 to the variation in the target reliability.

Stability under alternative order of solution

To demonstrate the stability of Equation 4.29, a different approach for the former part is presented here.

The strength reduction factor for the FRP rupture mode can be calculated according to Equation 4.29 by comparing it to a steel RC beam as the benchmark which results in Ø=0.70 if β_T=3.5 (**Table 4.3)**. Now, to calibrate the strength reduction factor of the other mode (concrete crushing), the FRP rupture mode is considered as the benchmark ($Ø_1$=0.70, λ_1=1.11 and β_T=3.5 in Equation 4.29) unlike the previous section which used a steel RC beam as the reference for both cases. If the results obtained by both approaches concur, in other words if a chain rule can be established for the comparative reliability, Equation 4.29 may be considered stable. This is necessary due to the approximate nature of this equation and the possibility of accumulating errors at each stage of the computation.

In this case δ_1 and δ_2 are relative to the two modes of failure (δ_1=0.157 and δ_2=0.158); therefore, β_c≈0 (Equation 4.22) and from Equation 4.27 it can be concluded that $Ø_1\lambda_2$=

$Ø_2\lambda_1$ where λ_1=1.11 and λ_2=1.19 (**Table 4.2**). With $Ø_1$=0.70, $Ø_2$ becomes equal to 0.75 which is consistent with the value given in **Table 4.3.**

4.7 VALIDATION OF THE PROPOSED METHOD

In the previous sections of this chapter, it is claimed that Equation 4.29 (or its more accurate form Equation 4.30) calibrates the strength reduction factor of element 2 so that it provides an acceptable fit for the reliability curves of the two elements, thus eliminating the need for repeating all computations for element 2. To put this claim to test and to understand the reason and consequences of behind the difference between the strength reduction factors obtained in this study (**Table 4.3,** under β_T=3.5) and those stipulated by ACI 440.1R-08 (**Table 4.2**), the reliability index is calculated for the entire range of the ratio of live load, *L*, to total load of dead plus live, *D*+*L*. To maintain consistency with Gulbrandsen (2005), the design load combination is assumed to be *U*=1.2*D*+1.6*L* and the values for statistical parameters of loads are taken from Nowak and Szerszen (2003) and given in **Table 4.4**, columns 2 and 3. It is also assumed that the design strength, *ØN*, is equal to the ultimate load or required strength, *U*, so there is no over or under-strength. The distribution of load variables is according to Ellingwood and Galambos (1982) as shown in **Table 4.4,** column 4.

Instead of using the approximate Equations 4.14 or 4.15 that assume both load and resistance conform to the lognormal distribution, the reliability index is calculated by the more sophisticated iterative Rackwitz-Fiessler method which takes into account different distributions (**Appendix D**). The results are plotted in **Fig. 4.1** which demonstrates that that the strength reduction factors of ACI 440.1R-08 achieve higher degrees of reliability than intended as represented by the two top curves. While the strength reduction factors proposed in this study (i.e., $Ø_1$=0.70 and $Ø_2$=0.75), target a reliability index approximately equal to that of a steel RC beam which in its turn is assumed to be approximately between 3.5 and 4.0**.** The two virtually inseparable reliability curves relative to the strength reduction factors derived in this study trail at a close distance the steel RC reliability curve as their target reliability, providing evidence for the validity of the comparative reliability method.

The validation may be further extended for other load combinations that include other load types, namely snow (*S*), earthquake (*E*) and wind (*W*) for which the reliability analyses of Gulbrandsen (2005) and subsequently ACI 440.1R-08 are silent. Although the method presented in this study does not provide the exact value of the reliability index, it guarantees that a safety performance comparable to that of a conventional RC may be expected. To demonstrate this quality, another load combination from ACI 318-11, e.g., U=1.2D+1.0L+1.6W may be considered. A combination containing wind load, while not necessarily critical for RC beams, is merely chosen because of the relative ease of applying the Rackwitz-Fiessler method to random variables that conform to the extreme value type I distribution (e.g., live load and wind load, **Table 4.4**). The presence of a third load component would necessitate a three dimensional representation of the reliability surface as opposed to the reliability curves given in **Fig. 4.1**. For simplicity, it is assumed that L=0 thus U=1.2D+1.6W, which makes a two dimensional representation possible. The statistical parameters of the wind load are shown in **Table 4.4**. **Fig. 4.2** shows the reliability index over the total range of the ratios of the wind load to the total load calculated by the Rackwitz-Fiessler numerical method. Again, the reliability curves associated with the safety factors calculated by the comparative reliability formulation (Equation 4.29) maintain their proximity to that of the steel RC beam, while those based on the strength reduction factors of ACI 440.1R-08 (the two top curves) indicate a more conservative design approach when compared to ACI 318-11, something which might be less than desirable. As for the case of **Fig. 4.1,** the two curves of the computed factors of $Ø_1$=0.70 and $Ø_2$=0.75 are to all effects overlapping.

4.8 SHEAR-CONTROLLED FAILURE OF FRP RC MEMBERS

ACI 440.1R-08 states that "The strength reduction factor of 0.75 given by ACI 318-05 for reducing nominal shear capacity of steel-reinforced concrete members should also be used for FRP reinforcement." which singles out this guideline among other codes of practice for proposing a larger strength reduction factor for shear as compared to flexure, and makes the need for validation more deeply felt. As always, the process of calibrating the safety factors starts by obtaining the statistical parameters of the element type in question. In this study, beams with or without transverse reinforcement (stirrups) are investigated independently as follows:

Statistical database: Test results compiled by Miano (2011) in combination with results from Matta et al. (2011) provide a statistical database for beams without stirrups which is summarized in **Table 4.5**. A similar database for beams with shear reinforcement (stirrups) is collected by Vitiello (2011) which is concisely displayed in **Table 4.6**.

FRP RC beams without stirrups: Based on the database summarized in **Table 4.5,** statistical parameters of FRP RC beams without stirrups under shear failure are calculated as λ_R=1.93 and δ_R=0.238 (**Table 4.7,** first row). **Appendix F** details the calculation of nominal shear capacity of such beams and their probabilistic parameters.

FRP RC beams with stirrups: Based on the database summarized in **Table 4.6** statistical parameters of FRP reinforced beams under shear failure are calculated as λ_R=1. 64 and δ_R=0.353 (**Table 4.7,** second row). Again, **Appendix F** provides an example of the calculation of nominal shear capacity of the RC beams with FRP stirrups.

For each of these two cases, Equation 4.30 may be used (especially for the second case where $\delta_R \geq 0.30$) to calculate the shear strength reduction factor. Substituting the probabilistic parameters of shear failure of a steel RC beam from **Table 4.1** ($Ø_1$=0.75, λ_1=1.23 and δ_1=0.109) and an FRP RC beam from **Table 4.7** (λ_2 and δ_2 from either of the first two rows), the strength reduction factor for the latter can be calculated for any presumed value of target reliability (the first two rows of **Table 4.8**) of which only the first column that corresponds to β_T=3.5 or the recognized safety level guaranteed by ACI 318, is of practical interest and the rest is only provided for comparison.

Evidently, the current shear safety factor of 0.75 is only justifiable for beam with no shear reinforcing, while in presence of such reinforcement, a drastic modification (from existing Ø=0.75 to no less than Ø=0.49) is necessary. Instead, this study advocates a modification to the limitations of shear design equation that eliminates the need for such a substantial drop in the strength reduction factor while maintains the desired level of safety.

Proposed modification to the FRP shear design equation limits

Redressing the issue of the strength reduction factor for beams with FRP stirrups requires pinpointing the source of deviation in resistance, as the sizeable coefficient of variation of such elements (δ_R=0.353) is the major contributor to the uncertainty associated with their strength that, subsequently, leads to a low safety factor.

Grouping the beams based on the level of shear reinforcement (V_f/V_c or FRP to concrete shear contribution) reveals a direct relationship between this value and deviation of resistance as the 32-member group of beams with $V_f \geq 3V_c$ was detected as the most temperamental (δ_R=0.614 from **Table 4.7**). Consistent with ACI 318 and ACI 440.1R approach, but using a lower threshold, it is proposed to limit shear reinforcement contribution, V_f, to $3V_c$. By imposing a new ceiling on the combination of the two components of the shear resistance, this modification excludes the set of elements with $V_f \geq 3V_c$ from the sample population:

$$V_n = V_c + V_f \;\text{ if }\; V_f \leq 3V_c \qquad (4.31)$$

$$V_n = 4V_c \;\text{ if }\; V_f > 3V_c \qquad (4.32)$$

Equation 4.32 is to cover those cases in which considerations such as achieving higher ductility through a tight arrangement of stirrups, supersede those of demand.

Leaving out these 32 cases from the sample population (**Table 4.6**), the statistical parameters are calculated for the remainder (**Table 4.7,** last row) with a considerable improvement in consistency of behavior (δ_R decreases from 0.383 to 0.226). The strength reduction factors are calculated anew using these fresh parameters which, when rounded, show compliance with the current code at the safety level of β_T=3.5 (**Table 4.8,** last row). Similar to the flexural case, **Fig. 4.3** and **4.4** show the reliability curves associated to this strength reduction factor (Ø=0.75 and $V_f \leq 3V_c$). The curves confirm that for both load combinations (U=1.2D+1.6L and U=1.2D+1.6W) and the limit state of shear, if $V_f \leq 3V_c$

then the reliability index of beams with FRP stirrups is comparable to that of steel RC beams.

4.9 DISCUSSION

The results of the comparative reliability analysis of beams internally reinforced with FRP bars measured against traditional steel RC beams may be summed up as follows:

- A flexural strength reduction factor of 0.70 is recommended for concrete beams internally reinforced with FRP bars when the failure mode is FRP rupture.
- A flexural strength reduction factor of 0.75 would be justifiable for concrete beams internally reinforced with FRP bars when the failure mode is concrete crushing. Although this is higher than the strength reduction factor of 0.65 imposed by ACI 318-11 on compression controlled sections, regardless of the reinforcement type, it is in agreement with the factor recommended by Nowak and Szerszen (2003).
- Even though the aforesaid strength reduction factors were derived based upon probabilistic consideration, they still submit to a traditional tendency of assigning a smaller strength reduction factor to the FRP rupture mode of failure, the mode that is presumed to be more sudden and brittle. Nevertheless, the considerable elongation of FRP bars up to the point of rupture which can be translated into exaggerated deflections, should FRP rupture govern the failure, provides enough warning prior to the abrupt collapse and may remove the need for differentiating between the two modes.
- Taking into account the three points above, due to the closeness of the two strength reduction factors a unified value of 0.70, irrespective of the failure mode, is recommended as it simplifies design and guarantees a higher level of safety.

• The current shear strength reduction factor of 0.75 may be maintained so long as the maximum effective level of shear resistance is dictated by Equations 4.31 and 4.32, in other words the nominal shear strength of a beam with FRP shear reinforcement must not exceed four times the strength provided by concrete ($V_n \leq 4V_c$ or equally $V_f \leq 3V_c$).

4.10 CONCLUSIONS

In the first part of this study, an alternative formulation of target reliability, β_T, is presented as a function of the resistances of two elements, unlike the conventional representation of the reliability index which is a function of load and resistance. Discarding the load parameters considerably simplifies calculations, as the need for obtaining the reliability index curves for numerous load combinations comprised of different load cases with different probability distributions is now circumvented. The relative ease of finding the probabilistic parameters of resistance, as compared to those of loads, as well as fewer uncertainties associated with resistance are additional advantages of the elimination of loads from the equations.

In the second part of the chapter, this comparative method is used to calibrate the strength reduction factors for design of concrete beams internally reinforced with FRP bars. As opposed to the traditional calibration methods which require a painstaking trial-and-error procedure of plotting the reliability curves for several load combinations over the conceivable ranges of proportions of load components, the proposed method obtains the strength reduction factor of the element of interest as an explicit function of the strength reduction factor, statistical parameters and an identical target reliability of the benchmark element. This is corroborated by the reliability curves of the benchmark elements (steel

RC beam) and those of the elements of interest (FRP RC beam), that demonstrate an acceptable fit between the two over a wide range of load ratios, if the strength reduction factors for the latter are derived according to the comparative formulation.

The closing discussion enumerates: a) modifications to the strength reduction factors of ACI 440.1-R08 associated with flexural elements (i.e., Ø=0.70 for both modes of failure); and b) limitations to the shear strength of beams with FRP stirrups (i.e.,$V_f \leq 3V_c$). These modifications ensure that a reliability level in compliance with the existing safety provisions is attainable or simply put, FRP RC beams are "as safe" and "as reliable" as the common steel RC beams.

The same procedure may be employed to calibrate the strength reduction factors associated with other RC elements, reinforced with FRP material such as columns or externally strengthened with FRP material.

Table 4.1: Strength reduction and statistical parameters of cast-in-place steel RC beams

Limit state	**Ø** [2]	$\boldsymbol{\beta_T}$ [3]	**Bias (λ)** [3]	**CoV(δ)** [3]
Flexure[1]	0.90	3.5	1.190	0.089
Shear	0.75	3.5	1.230	0.109

[1]Tension controlled

[2]ACI 318-11

[3] Nowak and Szerszen (2003)

Table 4.2: Strength reduction and statistical parameters of FRP reinforced beams subject to flexure

Limit state	**Ø** [1]	**Ø** [2]	$\boldsymbol{\beta_T}$ [2]	**Bias (λ)** [2]	**CoV(δ)** [2]
FRP rupture	0.55	0.65	3.5	1.11	0.157
Concrete crushing	0.65	0.65	3.5	1.19	0.158

[1]ACI 440.1R-08

[2]Gulbrandsen (2005)

Table 4.3: Calculated strength reduction factors for FRP RC beams subject to flexure for different target reliabilities

	Strength reduction factor (**Ø**)		
Limit state	β_T=3.5	β_T=4.0	β_T=4.5
FRP rupture	0.70	0.69	0.67
Concrete crushing	0.75	0.73	0.72

Table 4.4: Statistical parameters for load components

Load component	**Bias** (λ)[1]	**CoV**(δ)[1]	**Distribution**[2]
Dead load (cast-in-place)	1.05	0.10	Normal
Live load	1.00	0.18	Type I
Wind Load	0.78	0.37	Type I

[1]Nowak and Serszen (2003)
[2] Ellingwood and Galambos (1982)

Table 4.5: Experimental database of flexural elements without FRP stirrups(1)

Reference	No. of Specimens	f'_c ksi (MPa)	b in. (mm)	d_f in. (mm)	Longitudinal FRP: E_f ksi (GPa)	Longitudinal FRP: ρ_{fl} (%)	V_{exp} kips(kN)
Nagasaka et al. (1993)	2	3.3-5.1 (22.9-34.1)	9.8 (250)	10.1 (265)	8120 (56)	1.90	18.7-25.4 (83.0-113.0)
Tottori and Wakui(1993)	5	6.5-6.8 (44.6-46.9)	7.9 (200)	12.8 (325)	8410-27850 (58-192)	0.70-0.90	10.6-22.0 (47.0-98.0)
Maruyama and Zhao (1994)	4	4.0-5.1 (27.5-34.9)	5.9 (150)	9.8 (250)	13340 (92)	0.55-2.00	8.3-13.0 (37.0-57.8)
Maruyama and Zhao (1995)	2	5.0 (34.3)	5.9 (150)	9.8 (250)	15230 (105)	1.51-2.00	10.1-10.3 (45.0-46.0)
Nakamura and Higai(1995)	2	3.3-4.0 (22.7-27.8)	5.9 (150)	5.9 (150)	4210 (29)	1.30-1.80	7.4-8.1 (33.0-36.0)
Maruyama and Zhao (1996)	3	4.3-4.9 (29.5-34.0)	5.9-11.8 (150-300)	9.8-19.7 (250-500)	14500 (100)	1.07	6.4-31.5 (28.5-140.0)
Vijay et al. (1996)	1	6.5 (44.8)	7.9 (200)	10.4 (265)	7830 (54)	1.40	10.1 (45.0)
Duranovic et al. (1997)	4	3.8-5.5 (26.3-38.1)	5.9 (150)	8.3-8.7 (210-220)	6530 (45)	1.30	4.9-6.0 (22.0-26.5)
Mizukawa et al. (1997)	1	5.0 (34.7)	7.9 (200)	10.2 (260)	18850 (130)	1.30	13.9 (62.0)
Swamy et Aburawi(1997)	2	4.9-5.7 (34.0-39.0)	6.1-10.0 (154-254)	8.7 (222)	4930 (34)	1.55-1.60	4.4-8.8 (19.5-39.0)
Deitz et al. (1999)	5	3.9-4.5 (27.0-30.8)	12.0 (305)	6.2 (158)	5800 (40)	0.70	6.1-6.5 (27.0-29.0)
Alkhradji et al. (2001)	3	3.5 (24.1)	7.0 (178)	11.0-11.3 (279-287)	5800 (40)	0.77-2.00	8.1-12.0 (36.1-53.4)
Ospina, et al. (2001)	3	4.2-5.4 (28.9-37.5)	84.6 (2150)	4.7 (120)	4930 (34)	0.73-1.46	46.3-260 (206.0-
Yost et al. (2001)	19	5.3-5.5 (36.3-38.0)	7.0-12.0 (178-305)	7.6-8.9 (192-225)	5800-5950 (40-41)	0.36-2.00	6.0-11.5 (26.7-51.0)
Frosh (2002)	5	5.8-6.2 (39.8-42.6)	18.0 (457)	14.9 (379)	5510-7690 (38-53)	1.00-2.00	41.3-77.6 (183.7-
El Ghandour et al. (2003)	8	4.9-8.4 (34.0-58.0)	78.7 (2000)	6.9 (175)	6530-15950 (45-110)	0.15-0.30	38.2-71.3 (170.0-
Gross et al. (2003)	12	11.5 (79.6)	6.0-8.0 (152-203)	8.9 (225)	5800 (40)	1.25-2.00	6.8-10.9 (30.4-48.3)
Tariq and Newhook(2003)	11	4.9-6.3 (34.1-43.2)	5.1-6.3 (130-160)	12.2 (310)	6090-17400 (42-120)	0.70-1.50	9.7-13.0 (43.0-58.0)
Benmokrane (2004)	14	5.8-7.3 (40.0-50.0)	9.8-39.4 (250-1000)	6.1-12.8 (154-326)	5800-18850 (40-130)	0.39-2.00	13.5-42.7 (60.0-190.0)
Gross et al. (2004)	12	8.7-11.8 (60.3-81.4)	3.9-6.3 (89-159)	5.6 (141-143)	20160 (139)	0.33-0.76	2.0-5.2 (8.8-23.1)
Lubell et al. (2004)	1	5.8 (40.0)	17.7 (450)	38.2 (970)	5800 (40)	0.50	30.6 (136.0)
Razaqpur et al. (2004)	6	5.9-7.1 (40.5-49.0)	7.9 (200)	8.9 (225)	21030 (145)	0.25-0.88	8.1-10.6 (36.1-47.2)
El-Sayed et al. (2005)	10	6.3-9.1 (43.6-63.0)	9.8 (250)	12.8 (326)	5660-19580 (39-135)	0.87-2.00	13.5-39.1 (60.0-174.0)
El-Sayed et al. (2005)	8	5.8 (40.0)	39.4 (1000)	6.3-6.5 (159-165)	5800-16530 (40-114)	0.39-2.00	25.4-42.7 (113.0-
Guadagnini et al. (2005)	2	5.8-6.5 (40.3-44.9)	5.9 (150)	8.8 (223)	6530 (45)	1.28	6.1-10.0 (27.2-44.7)
Wegian and Abdalla (2005)	1	4.4 (30.0)	39.4 (1000)	6.4 (162)	6090 (42)	0.77	17.2 (76.5)
Ashour et al. (2006)	6	4.2-7.3 (28.9-50.2)	5.9 (150)	6.4-10.4 (163-263)	4640-5510 (32-38)	0.45-1.39	2.8-6.7 (12.5-30.0)
El-Sayed et al.(2006)	6	6.3-7.3 (43.6-50)	9.8 (250)	12.8 (326)	5800-18850 (40-130)	0.90-1.70	13.5-28.1 (60.0-125.0)
Kilpatrick et al. (2006)	9	7.0-13.3 (48.0-92.0)	16.5 (420)	2.9-3.0 (73-75)	6090 (42)	0.68-1.16	5.2-7.6 (23.1-33.9)
Kilpatrick and Easden (2006)	11	8.8-13.5 (61.0-93.0)	16.5 (420)	3.13.3 (79-83)	5800-6090 (40-42)	0.61-2.00	4.4-9.0 (19.5-40.0)

Continued on the next page

Table 4.5: Continued

Reference	No. of Specimens	f'_c ksi (MPa)	b_w in. (mm)	d in. (mm)	Longitudinal FRP E_f ksi (GPa)	Longitudinal FRP ρ_f(%)	V_{exp} kips(kN)
Valerio et al. (2006)	4	7.2 (49.8)	4.3 (110)	5.9 (150)	18850 (130)	0.93	4.8-5.6 (21.3-25.1)
Matta et al. (2011)	14	4.3-8.7 (29.5-59.7)	4.5-18.0 (114-457)	5.75-34.76 (146-883)	5950-6240 (41-43)	0.58-1.18	2.3-36.5 (10.1-162.2)
Total	196	3.3-13.5 (22.7-93.0)	3.50-84.60 (89-2150)	2.87-38.19 (73-970)	4060-27850 (28-192)	0.15-2.00	2.0-77.6 (8.8-345.0)

(1)Miano (2011) and Matta et al. (2011)

Table 4.6: Experimental database of flexural elements with FRP stirrups[1]

Reference	No. of Specimens[2]	f'_c ksi (MPa)	b in. (mm)	d_f or d_s in. (mm)	FRP bars: E_f or E_s ksi (GPa)	FRP bars: ρ_{fl} (%)	FRP stirrups: E_f ksi (GPa)	FRP stirrups: ρ_{fv} (%)	FRP stirrups: f_{fu} ksi(Mpa)	V_{exp} kips (kN)
Nagasaka et al. (1993)	29 (4)	3.3-5.8 (23.0-40.3)	9.84 (250)	9.96 (253)	8120-27270 (56-188)	0.31-1.90	6530-16680 (45-115)	0.50-1.50	71.1-133.4 (490-920)	36.4-82.3 (162.0-366.0)
Tottori and Wakaui (1993)	29 (11)	4.3-10.4 (29.4-71.6)	5.91-7.87 (150-200)	9.84-19.69 (250-500)	8410-29880 (58-206)	0.53-2.00	5220-19870 (36-137)	0.06-0.54	86.7-53.1 (598-1745)	13.0-51.8 (58.0-230.5)
Maruyama and Zhao (1994)	9	4.4-5.6 (30.5-38.3)	5.91 (150)	9.84 (250)	13630 (94)	0.55-2.00	13630 (94)	0.12-0.24	189.7 (1308)	13.3-26.9 (59.0-119.5)
Maruyama and Zhao (1995)	14	5.0 (34.3)	5.91 (150)	9.84 (250)	15230 (105)	1.51-2.00	5660-14500 (39-100)	0.42	159.5-188.5 (1100-1300)	12.2-28.3 (54.4-125.9)
Nakamura and Higai (1995)	8 (4)	4.8-5.2 (33.4-35.8)	7.87 (200)	9.84 (250)	4210-26100 (29-180)	1.61-1.72	4500 (31)	0.14-0.35	120.1 (828)	12.6-36.0 (56.0-160.3)
Vijay et al. (1996)	4	4.5-6.5 (31.0-44.8)	5.91 (150)	10.43 (265)	7830 (54)	0.64	7830 (54)	0.62-0.93	157.8 (1088)	25.9-28.6 (115.0-127.0)
Maruyama and Zhao (1996)	6	4.3-4.9 (29.5-34.0)	5.91-17.72 (150-450)	9.84-29.53 (250-750)	14500 (100)	1.07	4350 (30)	0.43-0.86	87.0 (600)	24.1-132.6 (107.0-590.0)
Alsayed et al. (1997)	4 (2)	5.1-5.7 (35.5-39.6)	7.87 (200)	12.20 (310)	5220-29000 (36-200)	0.97-1.37	6090 (42)	0.21-0.40	81.9 (565)	15.4-32.5 (68.5-144.4)
Duranovic et al. (1997)	2	4.6 (31.8)	5.91 (150)	8.66 (220)	6530 (45)	1.30	6530 (45)	0.35	108.8 (750)	11.0-15.0 (49.0-66.6)
Shehata et al. (1999)	8 (6)	4.8-7.8 (33.0-54.0)	5.31 (135)	18.50 (470)	19870-29000 (137-200)	1.25-1.32	5950-19870 (41-137)	0.24-1.40	92.8-250.9 (640-1730)	62.4-84.4 (277.5-375.5)
Whitehead and Ibell (2005)	5	8.1-9.3 (55.6-63.9)	4.33 (110)	3.94-7.87 (100-200)	8700 (60)	0.32	8700 (60)	0.19-0.80	203.1 (1400)	10.1-13.5 (45.0-60.0)
Total	118 (27)	3.3-10.4 (23.0-71.6)	4.33-17.72 (110-450)	3.94-29.53 (100-750)	4210-29880 (29-206)	0.31-2.00	4350-19870 (30-137)	0.06-1.50	71.1-253.1 (490-1745)	10.1-132.6 (45.0-590.0)

[1] Vitiello (2011)

[2] The number in parentheses indicates the number of specimens with longitudinal steel bars instead of FRP bars.

Table 4.7: Strength reduction and statistical parameters of FRP reinforced beams subject to shear

Shear reinforcement	**Ø** [1]	$\boldsymbol{\beta_T}$	**Bias (λ)**	**CoV(δ)**
V_f=0 (no stirrups)	0.75	3.5	1.93	0.238
no limit on V_f	0.75	3.5	1.64	0.353
$V_f > 3V_c$	0.75	3.5	1.22	0.614
$V_f \leq 3V_c$	0.75	3.5	1.80	0.226

[1] ACI 440.1R-08

Table 4.8: Calculated strength reduction factors for FRP RC beams subject to shear for different target reliabilities

	Strength reduction factor (**Ø**)		
Limit state	β_T=3.5	β_T=4.0	β_T=4.5
V_f=0 (no stirrups)	0.84	0.80	0.76
no limit on V_f	0.49	0.45	0.41
$V_f \leq 3V_c$	0.77	0.73	0.70

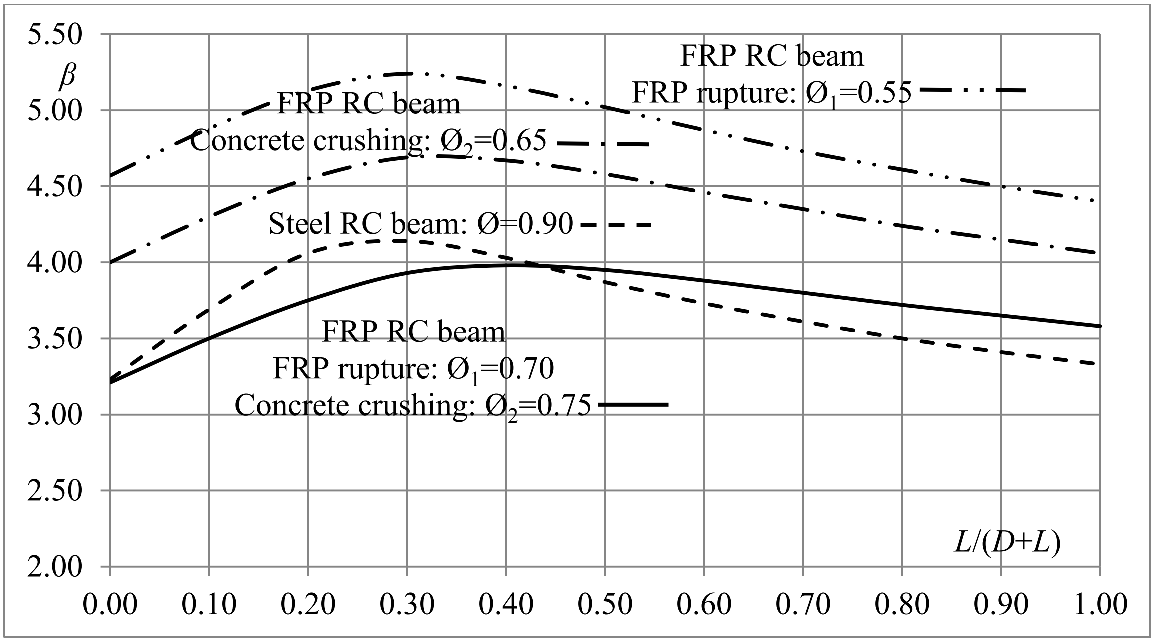

Figure 4.1: Reliability indices for beams made of ordinary concrete under flexure; load combination U=1.2D+1.6L

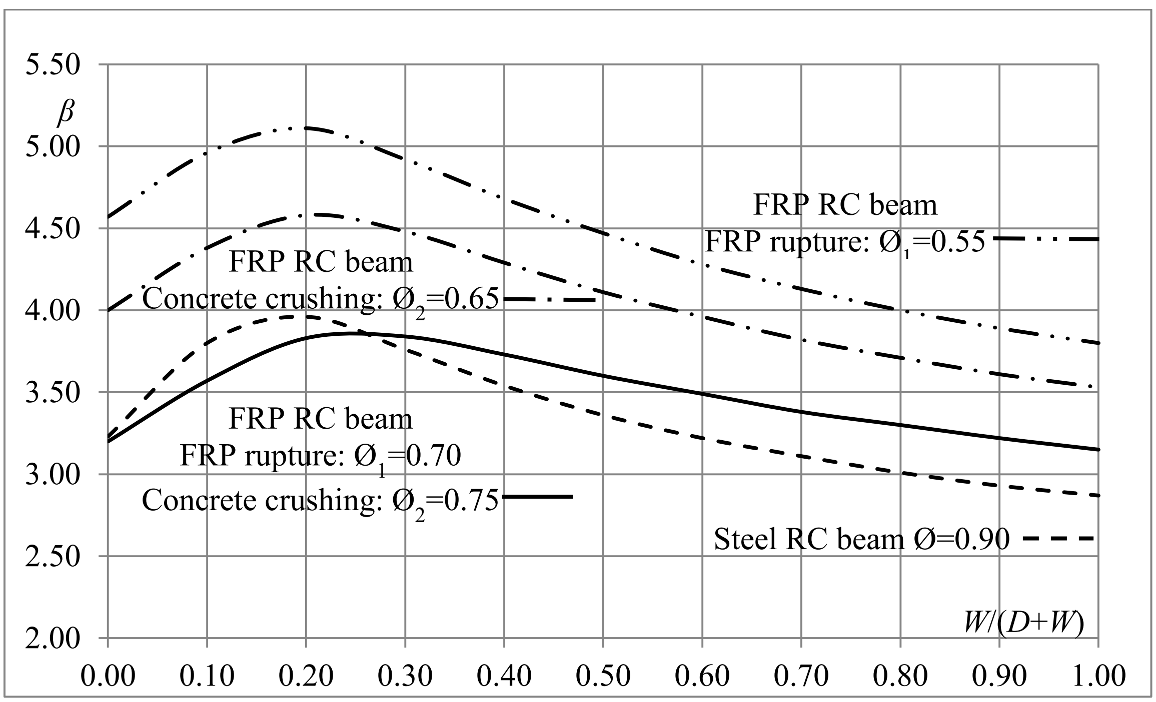

Figure 4.2: Reliability indices for beams made of ordinary concrete under flexure; load combination U=1.2D+1.6W

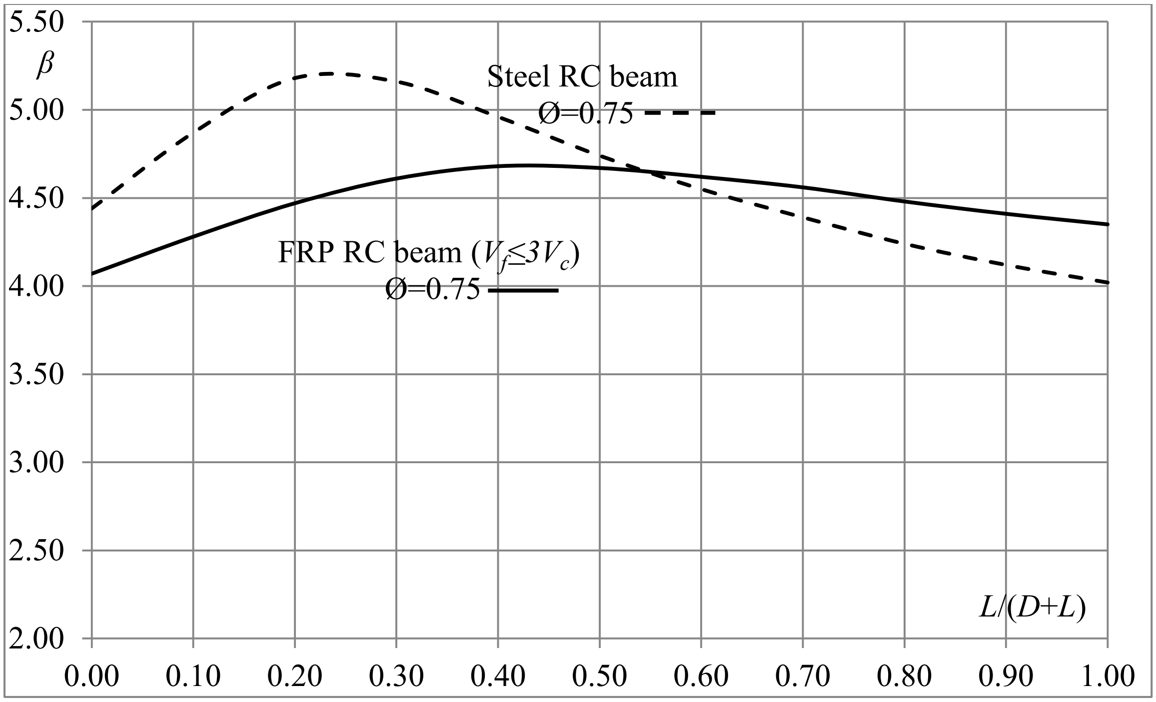

Figure 4.3: Reliability indices for beams made of ordinary concrete under shear; load combination U=1.2D+1.6L

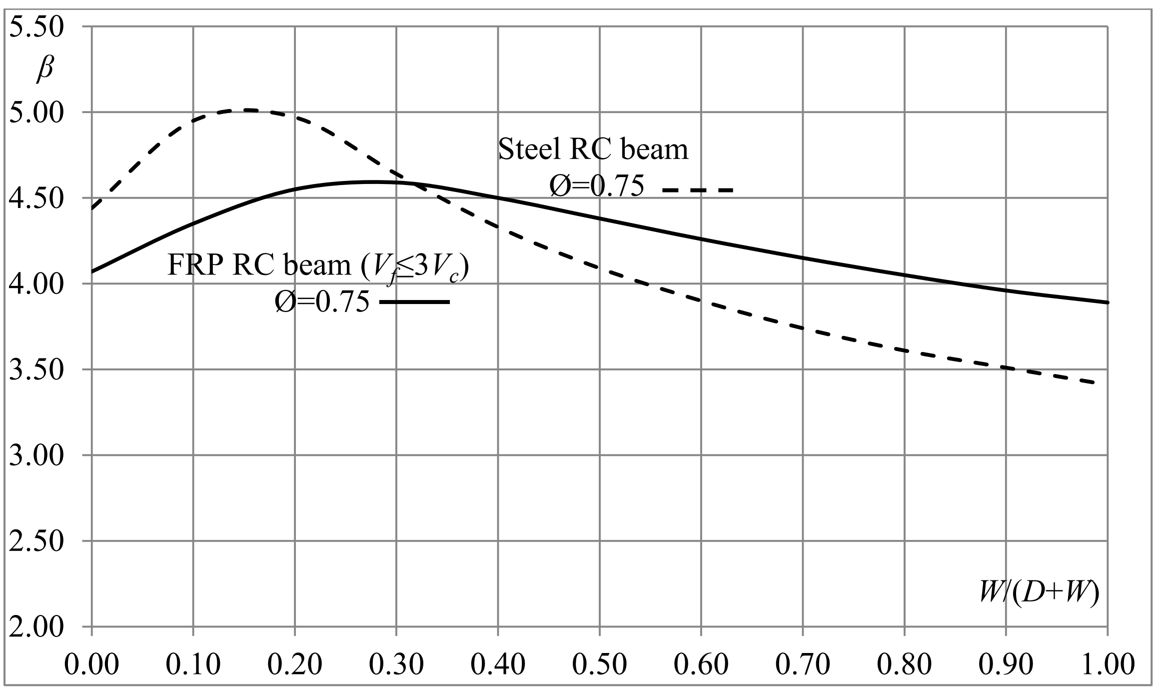

Figure 4.4: Reliability indices for beams made of ordinary concrete under shear; load combination U=1.2D+1.6W

CHAPTER 5

5. STUDY IV: STRENGTH REDUCTION FACTORS FOR FLEXURAL RC MEMBERS STRENGTHENED WITH NEAR-SURFACE-MOUNTED BARS

5.1 BACKGROUND

Assigning a strength reduction factor to RC members externally strengthened with FRP bars presents the code writers with a dilemma: on the one hand, any reduction factor has to address the very legitimate and yet, as will be explained later, frequently misconceived notions arising from adding to an RC members a new material, (e.g., FRP bars) whose behaviour exhibits more unpredictability than reinforcing steel. On the other hand, such a factor has to observe continuity, continuity being the sameness of the ultimate strength of an unstrengthened RC member with that of the same member with an infinitesimal amount of strengthening, a requirement which, needless to say, is violated should a smaller reduction factor be imposed on strengthened members. ACI 440.2R-08,"Guide for the Design and Construction of Externally Bonded FRP Systems for Strengthening Concrete Structures", has opted for an alternative solution: maintaining the strength reduction factors as stipulated by ACI 318, (e.g., Ø=0.90 for a tension-controlled flexure) and dictating a partial reduction factor to the contribution of FRP, e.g., ψ_f=0.85 for a section subject to pure bending. Historically, the primary reason for this approach was to allow for the use an "emerging" material system whose behaviour was not fully understood and proven. This method, although logical and effective, is today unsatisfactory, unsatisfactory as it constitutes a departure from ACI's approach of applying holistic reduction factors to the strength of members, and unsatisfactory because it resonates more as an "ignorance" factor than a safety factor, for it is designed mainly to alleviate non-quantified concerns and is derived chiefly by engineering judgement.

The seeming inescapability of a smaller reduction factor for FRP strengthened members stems from the belief that due to the higher randomness of the mechanical behaviour of FRP as compared to steel, and also the uncertainties associated with the installation of FRP, such member is necessarily more temperamental than an ordinary RC member, in other words adding FRP should lead to a build-up of uncertainties. This, however, might not always be true. Counterintuitive as it may sound, according to basic statistics, introducing a random variable into a system of random variables might very well reduce the deviation of the whole system, even if the newly added variable has a greater randomness than the former system. The simplest of such cases may be explained by two independent random variables with equal means and coefficients and variation. The sum of the two forms another random variable with a variation of about 70% of the variation of each of its components and, therefore, is more, not less, deterministic. When FRP strengthened RC members are concerned, the likelihood of such relaxations in uncertainty cannot be discounted.

It also should be noted that owing to the provisions of the guidelines (ACI 440), FRP components, compared to steel bars, commonly reserve a higher portion of their capacity between the point of their nominal strength to the point of their actual failure, i.e., rupture for FRP or yield for steel. In statistical terms FRP bars have a higher bias factor which can act as a pre-applied or hidden safety factor for them.

A combination of such arguments and other considerations related to the cost of repair, prompted the authors of this study to investigate the subject anew, for flexural members strengthened with near-surface mounted (NSM) FRP bars, by taking advantage of reliability analysis and computerized simulation techniques.

In this study, using a comprehensive test matrix of flexural members processed with the computerized Monte Carlo simulation technique, the probabilistic implications of applying FRP to RC beams and slabs, as near-surface-mounted (NSM) reinforcement, are investigated. The statistical data generated are then employed to recommend revised strength reduction factors for flexural RC members strengthened with NSM FRP bars, covered by ACI 440.2R-08, that are styled after the ACI 318 building code, do not compromise safety and yet do not impose needless restraints on benefiting from the full capacity of the strengthening system. By doing so not only a consistency among ACI documents can be achieved, but also repairs can be made satisfactorily safe and cost-effective at the same time.

5.2 OUTLINE

This study discusses the items below sequentially:

- The structural model of the strengthened RC members is discussed.
- The statistical model is defined based on the structural model and its variables and their descriptive parameters are introduced.
- The Monte Carlo simulation technique is briefly discussed.
- The statistical model is analysed via the Monte Carlo technique.

- The reliability analysis is employed to calculate the strength reduction factors for strengthened flexural members so that they achieve the same level of safety of ordinary RC beams and slabs.
- Recommendations about the modification of strength reduction factors are presented that allow for the elimination of the partial factors.

These steps break down the original unwieldy task of assessing the uncertainty of the complicated behaviour of a strengthened member into a series of smaller, more manageable questions of evaluating the uncertainties associated with each component and parameter contributing to the resistance of the member such as its materials, its dimensions, etc. None of these steps is absolutely judgment-free, yet anytime an engineering judgment is made, for want of information or statistics, it will be quantifiable and so will be its effects on the eventual outcome, making the methodology easily repeatable when richer experimental databases become available.

5.3 STRUCTURAL MODEL

The nominal flexural capacity of flexural RC members externally strengthened with FRP, of which NSM system is a subset, may be calculated as the sum of the strengths provided by each reinforcement component, the contribution of steel, M_{ns}, and the contribution of FRP, M_{nf}:

$$M_n = M_{ns} + M_{nf} \tag{5.1}$$

Where each term on the right of Equation 5.1 is a function of resistance variables calculated according to Chapter 10 of ACI 440.2R-08:

$$M_{ns} = f(A_s, f_y, f_c', b, d_s, \alpha_1, \beta_1, \varepsilon_{cu}, \varepsilon_{bi}) \tag{5.2}$$

$$M_{nf} = g(A_f, f_{fu}, \kappa_m, E_f, f_c', b, d_f, \alpha_1, \beta_1, \varepsilon_{cu}, \varepsilon_{bi}) \quad (5.3)$$

The symbols are defined in **Notations**. It should be noted the two components of resistance in Equation 5.1 are not totally decoupled, for, as a result of the repositioning of the neutral axis, M_{ns} declines ever so slightly as M_{nf} increases.

Modes of Failure

According to ACI 440.2R-08 two distinct modes of failure govern the behaviour of the externally FRP strengthened members and thus affect the calculation of Equations 5.1 to 5.3. The first mode is initiated by the crushing of the concrete in compression and occurs when the compressive strain in concrete reaches the maximum usable value of ε_{cu}=0.003. The debonding of the FRP bars initiates the second mode, which is similarly identified by the tensile strain of FRP system reaching the maximum level which can sustain the bond between the concrete and FRP bars. This maximum is referred to as the debonding strain or ε_{fd}, which is discussed later in this study. Reinforcing steel, also, may or may not have yielded at the point of failure, thus dividing each of the two aforesaid modes into two sub-modes, resulting in a total of four failure modes.

In this study, using the customary assumptions of the linearity of the plane sections in bending and compatibility of strains, the conditions that distinguish between these modes are derived based on the design parameters of the member. The reinforcement indices for steel and FRP, ω_s and ω_f, are defined as:

$$\omega_s = \frac{A_s f_y}{f_c' b d_s}; \; \omega_f = \frac{A_f f_{fd}}{f_c' b d_s} \quad (5.4)$$

Note that the denominator is the same for both parameters and f_{fd} is the stress in FRP corresponding to the debonding strain, ε_{fd}. The two modes can be predicted by comparison to the balanced condition:

$$\begin{cases} \omega_f \geq \omega_{fb}: \text{Concrete crushing} \\ \omega_f < \omega_{fb}: \quad \text{FRP debonding} \end{cases} \tag{5.5}$$

ω_{fb} depends on whether steel yields or not:

$$\omega_{fb} = \begin{cases} 0.85\beta_1 \frac{d_f}{d_s} \frac{\varepsilon_{cu}}{\varepsilon_{cu}+\varepsilon_{fd}+\varepsilon_{bi}} - \omega_s & : \quad \text{if steel yields.} \\ 0.85\beta_1 \frac{d_f}{d_s} \frac{\varepsilon_{cu}}{\varepsilon_{cu}+\varepsilon_{fd}+\varepsilon_{bi}} - \omega_s \frac{\left(\frac{d_s}{d_f}\right)(\varepsilon_{fd}+\varepsilon_{bi}) - \left(1-\frac{d_s}{d_f}\right)\varepsilon_{cu}}{\varepsilon_{sy}} & : \quad \text{if steel doesn't yield.} \end{cases} \tag{5.6}$$

ω_{sb}, separates the yielding sub-modes when concrete crushing is the case:

$$\begin{cases} \omega_s \leq \omega_{sb}: & \text{steel yields} \\ \omega_s > \omega_{sb}: & \text{steel doesn't yield} \end{cases} \tag{5.7}$$

Where:

$$\omega_{sb} = 0.85\beta_1 \frac{\varepsilon_{cu}}{\varepsilon_{cu}+\varepsilon_{sy}} - \omega_f \frac{\left(\frac{d_f}{d_s}-1\right)\varepsilon_{cu} + \left(\frac{d_f}{d_s}\right)\varepsilon_{sy} - \varepsilon_{bi}}{\varepsilon_{fd}} \tag{5.8}$$

In Equations 5.6 and 5.8, ε_{bi} is the strain level in the concrete substrate at the time of FRP installation and in practical cases varies, based on the loading, in the approximate range of 0.0005~0.001, while for the lab-made specimens normally ε_{bi}=0. For simplicity and uniformity, in this study the latter value is assumed for every case investigated, knowing that this assumption slightly over-predicts the resistance, if the member is deflected at the time of installation, but has less effect on the randomness of its behaviour which is the focus of this study.

Equations 5.4 to 5.8 confirm, as it could be expected, that the increase in steel reinforcing, ω_s, and FRP strengthening, ω_f, increase the likelihood of the first mode of failure, i.e., concrete crushing. Generally speaking, debonding is the dominant failure mode and concrete crushing is normally confined to the infrequent case of heavily-reinforced, heavily-strengthened beams.

5.4 STATISTICAL MODEL

In lieu of test results upon which the statistical parameters of the flexural capacity, partial (M_{ns} and M_{nf}) and total (M_n), can be established, these parameters are to be calculated from the combination of the factors that generate uncertainty in resistance. Each source of uncertainty is defined by its bias factor (λ, the ratio of the mean to the nominal value of a random variable), coefficient of variation (δ or CoV) and probabilistic distribution type. The following recounts how these data are collected from the literature or assumed by judgement.

Sources of uncertainty

Material factor: Although defined by its unique nominal or design value, the actual strength of a material is a random variable whose mean and standard deviation may be related to its nominal value from experimental data. **Table 5.1** lists the probabilistic identifiers of materials and each item is briefly discussed here:

f'_c: The compressive strength of concrete is not typically the controlling in the flexural capacity of a member, thus only the most common of concrete types is considered, the statistical parameters of which are taken from Novak and Szerszen (2003).

f_y: Grade 60 steel (f_y=60 ksi or 414 MPa) is the common type of reinforcing bars used in RC construction. Novak and Szerszen (2003) provides the statistical data.

f_{fu}: ACI 440 modifies the guaranteed tensile strength of FRP reinforcement, f^*_{fu}, by a reduction factor, $0.70 \leq C_E \leq 1.0$ based on the fiber type and conditions of environmental exposure:

$$f_{fu} = C_E f^*_{fu} \tag{5.9}$$

f_{fu}, the tensile strength of FRP bars or laminates is then a decisive factor in determining the flexural strength. In this study it is assumed that C_E regardless of its value is pre-applied and therefore only f_{fu} takes part in the computations. Ignoring a potential source of uncertainty in C_E is deemed to be mitigated by ACI 440.2R-08 statement that its recommended values are "conservative estimates". In other words, in reality FRP material might be slightly more volatile and yet in contrast slightly stronger than what this study presumes.

Unlike steel and to some extent concrete producers that comply with well-established standards, FRP manufacturers protocols are less controllable. This leads to a variation in the mechanical characteristics of FRP products which in turn, makes acquiring an objective assessment of the probabilistic parameters of FRP materials a challenging task, as the test samples are usually biased towards the products and manufacturers they represent. In this study, it is assumed that FRP products exhibit a level of consistency of characteristics higher than that of concrete and yet lower than steel, which is the most predictable of construction materials. This assumption confines FRP's coefficient of variation between those of concrete and steel (i.e., $0.05 \leq \delta \leq 0.10$). For FRP bars, circular or rectangular, δ=0.08 is selected as a reasonable estimate. A bias factor of λ=1.20 that corresponds to such a deviation, according to Gulbrandsen (2005), completes the couple. Two generic types of FRP are assumed with type 1 having significantly higher tensile strength and modulus of elasticity than type 2.

E_f: The statistical data is according to Gulbrandsen (2005).

κ_m: FRP debonding is the dominant failure mode of the strengthened members. The debonding strain and stress of the NSM reinforcement, ε_{fd} and f_{fd} , are defined as:

$$\varepsilon_{fd} = \kappa_m \varepsilon_{fu}\ ;\ f_{fd} = \kappa_m f_{fu} \tag{5.10}$$

For NSM reinforcement, ACI 440.2R-08 states that 0.60≤κ_m≤0.90, "depending on many factors such as member dimensions, steel and FRP reinforcement ratios, and surface roughness of the FRP bar" and recommends the use of κ_m=0.70. Assuming a simple uniform random distribution over that range, the statistical parameters are estimated. The coefficient of variation of 0.115 makes the debonding factor, κ_m, the major source of material uncertainty.

Fabrication Factor: The variations in dimensions and geometry fall under the category of the fabrication factor. These parameters as shown in **Table 5.2** are generally based on Novak and Szerszen (2003), except for those discussed below:

c: The concrete cover to steel reinforcement center is assumed to be statistically independent of the effective depth of steel bars, but with the same probabilistic parameters of bias and variation.

d_f: The effective depth of external FRP, with negligible approximation, is estimated as:

$$d_f = d_s + c \tag{5.11}$$

Therefore, d_f is a random variable whose nominal and mean value and its standard deviation depend upon *c* and *d* and their relative magnitude. Also note that higher variation and smaller bias factor for the effective depth and concrete cover differentiate slabs from beams.

b (for slabs):The unit width of strip used in the analysis and design of the slabs is assumed to be a deterministic parameter.

A_f: The statistical data for the FRP bars is according to Gulbrandsen (2005).

Professional factor: is the ratio of the actual to theoretical behavior, and is applied to steel and FRP contributions independently. In mathematical terms, these factors (material, fabrication and professional) boil down to the statistical model of:

$$R = R_s + R_f \tag{5.12}$$

Where, *R,* the total flexural strength of the member is a random variable comprised of steel and FRP random contributions, R_s and R_f:

$$R_s = P_s M_s; R_f = P_f M_f \tag{5.13}$$

M_s and M_f are random variables whose nominal values are M_{ns} and M_{nf} (Equations 5.2 and 5.3), and contain the collective effect of the material and fabrication factors. P_s and P_f are the random variables representing the professional factor and account for the uncertainties associated with analysis parameters (ε_{cu}, α_1 and β_1) that are presumed to be deterministic. The probabilistic parameters of P_s (i.e., λ and δ) are according to Novak and Szerszen (2003). The same parameters are deemed by the authors to be applicable to P_f; however, to remain reasonably conservative the bias factor (λ) for FRP is lowered to 1.00 (**Table 5.3**).

5.5 SIMULATION MATRIX

Table 5.4 shows a summary of the simulation matrix. Thirty two sets of slabs and thirty sets of beams, divided equally between the two types of FRP specified in **Table 5.1**, are simulated. Each set contains 5 members, one unstrengthened member and 4 with different levels of strengthening applied to that, resulting in a total of 310 simulated members. All the unstrengthened members, that represent the common designs, are tension controlled in flexure, according to ACI 318-11. This can be easily confirmed by the limits of steel ratios, ρ_s. The focus is, of course, on the flexure and other types of resistance, shear and torsional, are assumed to be sufficient.

A very limited number of simulations pertain to the strengthening range of 100-150%; however, the utmost level of strengthening is generally restricted to approximately 100%, which means that the nominal flexural capacity may, as a maximum, be nearly doubled by strengthening. This is consistent with the limit imposed by ACI 440.2R-08, Chapter 9.

5.6 MONTE CARLO SIMULATION

Monte Carlo method technique is employed to generate samples of numerical data (e.g., resistance) from which the statistical parameters such as mean and standard deviation, or equally bias factor and coefficient of variation, may be calculated. **Appendix G** provides an example of how this technique is employed in this study. **Tables 5.5** and **5.6** contain examples of the results of such simulations. For each design, in addition to the nominal values of resistance, the statistical parameters are also calculated by the Monte Carlo simulation.

5.7 RELIABILITY ANALYSIS AND CALCULATION OF STRENGTH REDUCTION FACTORS

The strength reduction factors are calibrated based on the comparative reliability equation elicited from Chapter 4:

$$\frac{\ln\left[\frac{\phi_{NSM}}{\phi_{RC}}\frac{\lambda_{RC}}{\lambda_{NSM}}\right]}{\sqrt{\delta_{NSM}^2+\delta_{RC}^2}} = \frac{\delta_{RC}-\delta_{NSM}}{\delta_{NSM}+\delta_{RC}}\beta_T \tag{5.14}$$

Equation 5.14 relates the strength reduction and statistical parameters (Ø, λ and δ) of an NSM member, denoted by the subindex *NSM,* to those of an ordinary RC member with the subindex *RC,* which acts as the benchmark, if the same level of safety or target reliability of β_T is expected from both member types. In this study, the benchmark RC members are assumed to have the same statistical characteristics of resistance of the member prior to strengthening. **Tables 5.5** and 5.**6** can be recalled to clarify the subject. These tables display sample simulated sets of five members. The first row of each set, i.e., the unstrengthened or the ordinary RC member constitutes the benchmark member. The benchmark reduction factor for each member is calculated according to ACI 318-11 and the safety levels of β_T=2.5 for slabs or β_T=3.5 for beams are selected according to Novak and Szerszen (2003). The failure mode predicted by the design (nominal) values is denoted by mode I for concrete crushing and mode II for debonding. The last column is the ratio of the reduction factor of the strengthened member, $Ø_{NSM,}$ to that of an RC member, $Ø_{RC}$, calculated according to Equation 5.14.

5.8 DISCUSSION

Investigation of the results

A general observation can be made from the results of the simulations, represented partially by **Tables 5.5** and **5.6**: the FRP contribution has a higher deviation and higher bias factor than the steel contribution. This, nevertheless, is a forgone conclusion due to the higher bias and deviation associated with FRP as a material. Further generalizations may be drawn by a more in-depth look at the results:

Slabs: The variation of the strength of the strengthened member is normally lower than the variations of each of its components. This is mainly attributable to the variation reduction mentioned in **section 5.1** and accounts, partially, for the increased reduction factors compared to the unstrengthened members (cases with $Ø_{NSM}$ / $Ø_{RC}$ >1.0).

Beams: The fabrication factors of beams, as opposed to those of slabs, have a markedly lower randomness, which results in the more deterministic behavior of the beams which is reflected in their lower coefficients of variation. In the case of beams, the addition of FRP generally increases the variation when the failure is governed by concrete crushing. For debonding failures, the trend is mainly similar to that of slabs which undergo the same failure mode.

Aside from these general statements, a pattern must be detected in the calculated strength reduction factors so that they can be formulated for practical use.

When steel yields, Equations 5.5 and 5.6 can be easily reworked to predict the failure mode based on the ratio of the collective reinforcement index, $\omega_s+\omega_f$, to the parameter, ω_b, defined in this study as:

$$\omega_b = 0.85\beta_1 \frac{d_f}{d_s}\frac{\varepsilon_{cu}}{\varepsilon_{cu}+\varepsilon_{fd}+\varepsilon_{bi}} \tag{5.15}$$

ω_b marks the combination of the steel and FRP indices which results in the simultaneous FRP debonding and concrete crushing, if steel has already yielded. If $(\omega_s+\omega_f)/\omega_b \leq 1.0$, the debonding mode governs the failure; else, failure is initiated by the concrete crushing. This ratio can also be interpreted as an indicator of the deflection before failure. The growth of $(\omega_s+\omega_f)/\omega_b$ leads to smaller deflections before the flexural failure and vice versa. Therefore to formulate the Ø factor, members were regrouped based on this ratio. **Fig. 5.1** and **5.2** display the reduction factors calculated against the strengthening levels for two of such groups, i.e., $1.0\leq(\omega_s+\omega_f)/\omega_b\leq2.0$ and $2.0\leq (\omega_s+\omega_f)/\omega_b\leq4.0$,. It is observed that for the group with $(\omega_s+\omega_f)/\omega_b\leq1.0$, or those that fail by debonding, the calculated strength reduction factor is almost always larger than $\emptyset_{RC}$. One exception is presented in the last row of **Table 5.5** which is, admittedly, a design far from normal. As shown graphically, by the totally conservative approach of finding the lower boundary of the points a reduction factor can be suggested as:

$$\frac{\emptyset_{NSM}}{\emptyset_{RC}} = \begin{cases} 1.0 & : \quad \text{if } \frac{\omega_s+\omega_f}{\omega_b} \leq 1.0 \\ 1-\frac{\Delta}{9} \geq \frac{8}{9} & : \quad \text{if } \frac{\omega_s+\omega_f}{\omega_b} \geq 2.0 \end{cases} \tag{5.16}$$

Where, Δ, is the strengthening level:

$$\Delta = \frac{M_n|_{\text{strengthened}} - M_n|_{\text{unstrengthened}}}{M_n|_{\text{unstrengthened}}} \tag{5.17}$$

And for values of $1.0 \leq (\omega_s+\omega_f)/\omega_b \leq 2.0$ a linear interpolation is satisfactory.

Example

As an example, the procedure of the calculation of the ultimate flexural strength, $M_{u=}\emptyset M_n$, for one of the beams in **Table 5.6**(Set 2, fourth row) is detailed here:

Flexural capacity, $M_{n_{\text{strengthened}}}$, calculations:

From the beam and material properties (**Tables 5.1** and 5.**6**) the reinforcement indices (Equation 5.4) can be calculated as: ω_s=0.1500, ω_f=0.1524, assuming that ε_{bi}=0.00.

Equations 5.7 and 5.8 are used to check if the steel yields:

ω_{sb} = 0.3857>ω_s; therefore, steel yields.

Debonding can be checked by equations:

ω_b=0.1865, $(\omega_s+\omega_f)/\omega_b$=1.62>1.0; no debonding is expected and concrete crushing is the failure mode.

In this study the design equations governing this mode of failure (concrete crushing and steel yielding) are derived as:

Stress ratio in FRP:

$$f=\frac{f_f}{f_{fd}}=\frac{\sqrt{\left(\frac{\omega_s}{\omega_f}-\frac{\varepsilon_{cu}+\varepsilon_{bi}}{\varepsilon_{fd}}\right)^2+\frac{3.4\beta_1}{\omega_f}\frac{\varepsilon_{cu}}{\varepsilon_{fd}}\frac{d_f}{d_s}}-\left(\frac{\omega_s}{\omega_f}+\frac{\varepsilon_{cu}+\varepsilon_{bi}}{\varepsilon_{fd}}\right)}{2}\leq 1 \quad (5.18)$$

f_f is the stress in FRP. The steel contribution is calculated as:

$$M_{ns}=f'_c\omega_s\left(1-\frac{\omega_{s+}f\omega_f}{1.7}\right)bd_s^2 \quad (5.19)$$

And the contribution of FRP:

$$M_{nf}=f'_cf\omega_f\left(\frac{d_f}{d_s}-\frac{\omega_{s+}f\omega_f}{1.7}\right)bd_s^2 \quad (5.20)$$

Substituting the corresponding values into Equations 5.18 to 5.20:

f=0.667, M_{ns}=1473.7 kip.in (166.5 kN.m), M_{nf}=1187.8 kip.in (134.2 kN.m), and according to Equation 5.1: M_n=2661.5 kip.in (300.7 kN.m).

Strength reduction factor according to ACI 440.2:

$$M_u=\emptyset_{RC}\left(M_{ns}+\psi_fM_{nf}\right) \quad (5.21)$$

Where:

$$0.65\leq\emptyset_{RC}=0.65+0.25\frac{\varepsilon_s-\varepsilon_{sy}}{0.005-\varepsilon_{sy}}\leq 0.90;\ \psi_f=0.85 \quad (5.22)$$

ε_s is the strain in steel reinforcement which can be calculated from the compatibility of strains:

$$\varepsilon_s=\frac{d_s}{d_f}\left(f\varepsilon_{fd}+\varepsilon_{bi}\right)-(1-\frac{d_s}{d_f})\varepsilon_{cu}\geq\varepsilon_{sy} \quad (5.23)$$

ε_s=0.0056; therefore, $\emptyset_{RC}$=0.90 and M_u=2235.0 kip.in (252.6 kN.m).

Strength reduction factor according to this study:

The strength reduction factor is calculated from Equation 5.16:

$(\omega_s+\omega_f)/\omega_b$=1.62>1.0; therefore, $8/9\leq\emptyset_{NSM}/\emptyset_{RC}\leq 1.0$ and the strengthening level, Δ, must be computed.

From conventional reinforced concrete design:

$$M_n|_{\text{unstrengthened}} = f'_c \omega_s \left(1 - \frac{\omega_s}{1.7}\right) b d_s^2 = 1577.2 \text{ kip.in (178.2 kN.m)} \tag{5.24}$$

And according to Equation 5.17 the strengthening level is: Δ=0.69. From Equation5.16:

if $(\omega_s+\omega_f)/\omega_b$=2.0; then, $Ø_{NSM}/Ø_{RC}$=1.0-(0.69)/9=0.923

The strength reduction factor is obtained by interpolation:

$Ø_{NSM}/Ø_{RC}$=(1.62-1.00)(0.923)+(2.00-1.62)(1.0)=0.952 (compare to the value of 0.962 obtained by the reliability analysis, i.e., Equation 5.14 and **Table 5.6.**)

$Ø_{RC}$=0.90; therefore, $Ø_{NSM}$=0.857

And, eventually, $M_u=Ø_{NSM}M_n$=2281.0 kip.in (257.7 kN.m), slightly higher than ACI 440.2 recommendation.

Table 5.7 summarizes similar calculations for the beams of **Table 5.6.**

Comparison with ACI 440.2

A direct comparison between the strength reduction factors obtained in this study (Equation 5.16) with that stipulated by ACI 440.2R-08 (Equation 5.22) might not seem very streamlined. Nevertheless, if the question is approached from a practical point of view the issue can be greatly simplified. The lower values of the current reduction factors from ACI 440.2 are relatively of low practical importance due to a series of reasons, including:

- They require low levels of strain on the tension side of the flexural member which can be translated into low levels of stress in the strengthening system and its decreased efficiency. Such a situation may defeat the idea of strengthening in the first place.

- The overwhelming majority of the members in need of strengthening are lightly reinforced, either because of their nature, like slabs, or because of the reason that originates the requirement of repair, like under-designed beams. Such designs, always, lead to large deformations on the tension side.
- Mainly due to the installation hardships, the additional strength gained by adding FRP hardly ever goes beyond 50%, with the normal values revolving around 20 to 25%. This again means less restraint for the member and comparatively large deflections on the tension side.

Tables 5.6 and **5.7** portray these arguments in a more tangible form: Set 1 is composed of a lightly-reinforced RC beam (ρ_s=0. 5% slightly higher than the minimum permissible by ACI 318, ρ_{min}=0. 35%) with different degrees of strengthening, Δ. Set 3, on the other hand is heavily reinforced (ρ_s=1.5%) and, unlike Set 1, for the strengthening levels of **Table 5.7** requires $\O_{RC}$ factors of smaller than 0.90, according to ACI 440.2. A comparison between the relative ratios of FRP to steel (A_f/A_s in **Table 5.6**, or more comprehensively ω_f/ω_s in **Table 5.7**) confirms the much higher efficiency of the strengthening in Set 1. The absence of any slabs and the rarity and the relative impracticality of the beams that require a factor lower than 0.90 by ACI 440.2 in the simulated designs confirm that $\O_{RC}$=0.90 may be regarded as the most applicable strength reduction factor recommended by the guideline, which, certainly, has to be applied in combination with its partial reduction factor of ψ_f=0.85 imposed on the FRP contribution.

5.9 CONCLUSIONS

Based on this discussion, a simple and general comparison between the two proposed factors for NSM systems, can now made:

- For lightly reinforced and strengthened flexural members, this study recommends a reduction factor of 0.90 and eliminates the ACI's partial factor of 0.85.
- For other cases, this study recommends a variable but unified reduction factor that, roughly speaking, has the equal effect of the double factor of ACI 440.2.

The obvious advantages can be restated as the increased cost-effectiveness without risking the safety and restoring the tradition of ACI in applying the reduction factors to the overall strength of a member and not to the individual materials.

This study concentrates on the NSM systems; however, other types of installation, such as pultruded plates and in-situ lay-up and other limit states such as shear and axial force may be investigated in the same manner, provided that the data concerning their uncertainties are reliably provided or estimated.

Table 5.1: Statistical parameters for materials

Material		Property	Nominal value	Bias (λ)	CoV(δ)	Distribution
Concrete[1]		f'_c ksi (MPa)	4.0 (27.6)	1.24	0.10	lognormal
Steel bars[1]		f_y ksi (MPa)	60 (414)	1.145	0.05	lognormal
FRP bars[2]	1	f_{fu} ksi (MPa)	250 (1725)	1.20[2]	0.08[2]	lognormal
		E_f ksi (GPa)	20000(138)	1.04[2]	0.08[2]	lognormal
	2	f_{fu} ksi (MPa)	90 (620)	1.20[2]	0.08[2]	lognormal
		E_f ksi (GPa)	6000(41)	1.04[2]	0.08[2]	lognormal
		κ_m	0.70	1.071	0.115	uniform

[1] Nowak and Szerszen (2003)
[2] Gulbrandsen (2005)

Table 5.2: Statistical parameters for dimensions of concrete, steel and FRP

Item	Member	Bias (λ)	CoV(δ)	Distribution
d_s[1]	beam	0.99	0.04	lognormal
	slab	0.92	0.12	
c	beam	0.99	0.04	lognormal
	slab	0.92	0.12	
b	beam[1]	1.01	0.04	lognormal
	slab	1.00	0.00	deterministic
A_s[1]	beam, slab	1.00	0.015	lognormal
A_f[2]	beam, slab	1.00	0.03	lognormal

[1] Nowak and Szerszen (2003)
[2] Gulbrandsen (2005)

Table 5.3: Statistical parameters of professional factors

Item	**Nominal value**	**Bias (λ)**	**CoV(δ)**	**Distribution**
P_s(1)	1.00	1.02	0.06	lognormal
P_f	1.00	1.00	0.06	lognormal

(1)Nowak and Szerszen (2003)

Table 5.4: Summary of simulated members

Member	$\boldsymbol{d_s}$ in. (mm)	$\boldsymbol{d_f}$ in. (mm)	$\boldsymbol{b}$ in. (mm)	$\boldsymbol{\rho_s}$ (%)	**Strengthening Level** (%)	**Number of members**
Slabs	2.5-10.5 (64-267)	4.0-12.0 (102-305)	12.0 (305)	0.2-0.5(1)	20-110(3)	160
Beams	9.5-27.5 (241-699)	12.0-36.0 (305-914)	8.0-24.0 (203-610)	0.5-1.5(2)	20-150(3)	150

(1)$\rho_s = A_s/bd_f$
(2)$\rho_s = A_s/bd_s$
(3)Approximate range.

Table 5.5: Examples of results for NSM strengthened slabs per unit width of 1.0 ft. (305 mm)

Set	Slab Properties[1] d_f in. (mm)	d_s in. (mm)	A_s in.^{2} (mm^2)	A_f[2] in.^{2}	Steel contribution M_{ns}[5] kip.in	λ	δ	FRP contribution M_{nf}[5] kip.in	λ	δ	Total strength (NSM) M_n[5] kip.in	λ	δ	Strengthening level (%)	Failure mode[3]	$Ø_{NSM}/Ø_{RC}$[4]
1 (ρ_s=0.2%)	6.0 (152)	4.5 (114)	0.144 (93)	0.00	38.0	1.090	0.188	0.0	-	-	38.0	1.090	0.188	0	-	1.000
				0.01	37.2	1.085	0.189	10.2	1.205	0.200	47.4	1.111	0.175	25	II	1.043
				0.02	37.1	1.086	0.191	20.3	1.206	0.200	57.3	1.128	0.172	51	II	1.064
				0.03	36.9	1.086	0.190	30.3	1.205	0.198	67.2	1.139	0.170	77	II	1.079
				0.04	36.8	1.085	0.190	40.3	1.207	0.200	77.1	1.149	0.172	103	II	1.083
2 (ρ_s=0.3%)	8.0 (203)	6.5 (165)	0.288 (186)	0.00	108.7	1.087	0.176	0.0	-	-	108.7	1.073	0.176	0	-	1.000
				0.02	**106.8**	**1.087**	**0.174**	**26.9**	**1.207**	**0.199**	**133.6**	**1.111**	**0.163**	**23**	**II**	**1.059**
				0.04	106.2	1.085	0.177	53.5	1.206	0.202	159.8	1.125	0.165	47	II	1.070
				0.06	105.7	1.085	0.177	80.0	1.205	0.200	185.7	1.137	0.164	71	II	1.081
				0.08	105.2	1.082	0.177	106.2	1.201	0.198	211.4	1.142	0.165	95	II	1.084
3 (ρ_s=0.4%)	10.0 (254)	8.5 (216)	0.480 (307)	0.00	234.6	1.087	0.167	0.0	-	-	234.6	1.077	0.167	0	-	1.000
				0.04	230.8	1.086	0.169	66.6	1.207	0.201	297.3	1.113	0.160	27	II	1.047
				0.08	229.1	1.083	0.170	132.3	1.201	0.198	361.4	1.126	0.160	54	II	1.059
				0.12	227.4	1.079	0.172	197.3	1.187	0.200	424.7	1.129	0.165	81	II	1.052
				0.16	225.6	1.080	0.175	261.4	1.161	0.204	487.0	1.123	0.174	108	II	1.031

Continued on the next page

Table 5.5: Continued

Set	d_f in. (mm)	d_s in. (mm)	A_s in.^{2} (mm^2)	A_f[(2)] in.^{2}	A_f[(2)] in.^{2}	M_{ns}[(5)] kip.in	λ	δ	M_{nf}[(5)] kip.in	λ	δ	M_n[(5)] kip.in	λ	Strengthening level (%)	Failure mode[(3)]	$Ø_{NSM}/Ø_{RC}$[(4)]
Slab Properties[(1)]					**Steel contribution**			**FRP contribution**			**Total strength (NSM)**					
4 (ρ_s=0.5%)	12.0 (305)	10.5 (267)	0.720 (465)	0.00	430.7	1.086	0.163	0.0	-	-	430.7	1.080	0.163	0	-	1.000
				0.06	424.4	1.086	0.168	118.9	1.203	0.203	543.3	1.112	0.160	26	II	1.036
				0.12	420.6	1.081	0.170	236.0	1.180	0.205	656.5	1.117	0.165	52	II	1.031
				0.18	416.6	1.078	0.169	351.0	1.136	0.209	767.6	1.105	0.172	78	II	1.008
				0.24	412.2	1.075	0.172	463.7	1.077	0.224	875.9	1.076	0.186	103	II	0.958

(1)Other properties according to **Table 5.1**.
(2)FRP type 1 from **Table 5.1**.
(3)I: concrete crushing, II: debonding
(4)According to Equation 5.14 with β_T=2.5
(5)1.0 kip.in=0.113 kN.m

Table 5.6: Examples of results for NSM strengthened beams

Set	Beam properties[1]					Steel contribution			FRP contribution			Total strength (NSM)			Strengthening level (%)	Failure mode[3]	$Ø_{NSM}/Ø_{RC}$ [4]
	b in. (mm)	d_f in. (mm)	d_s in. (mm)	A_s in.^{2} (mm^2)	A_f[2] in.^{2}	M_{ns}[5] kip.in	λ	δ	M_{nf}[5] kip.in	λ	δ	M_n[5] kip.in	λ	δ			
1 (ρ_s=0.5%)	8.0 (203)	12.0 (305)	9.5 (241)	0.38 (245)	0.00	207.0	1.165	0.095	0.0	-	-	207.0	1.159	0.095	0	-	1.000
					0.075	204.3	1.164	0.095	54.2	1.292	0.161	258.5	1.189	0.087	26	II	1.049
					0.15	203.0	1.163	0.096	107.7	1.288	0.158	310.7	1.204	0.089	52	II	1.054
					0.225	201.6	1.162	0.097	160.8	1.272	0.150	362.4	1.208	0.092	78	II	1.052
					0.30	200.2	1.160	0.098	213.2	1.246	0.143	413.4	1.202	0.094	103	II	1.040
2 (ρ_s=1.0%)	12.0 (305)	18.0 (457)	15.5 (394)	1.86 (1200)	0.00	1577.2	1.161	0.093	0.0	-	-	1577.2	1.170	0.093	0	-	1.000
					0.60	1529.1	1.173	0.095	569.4	1.080	0.169	2098.5	1.148	0.091	33	I	0.986
					1.20	1497.9	1.176	0.095	922.4	1.098	0.167	2420.4	1.147	0.097	53	I	0.971
					1.80	**1473.7**	**1.178**	**0.096**	**1187.8**	**1.102**	**0.161**	**2661.5**	**1.144**	**0.099**	**69**	**I**	**0.962**
					2.40	1453.6	1.178	0.095	1402.6	1.104	0.160	2856.2	1.142	0.102	81	I	0.953
3 (ρ_s=1.5%)	18.0 (457)	24.0 (610)	21.5 (546)	5.805 (3745)	0.0	6497.3	1.171	0.094	0.0	-	-	6497.3	1.171	0.094	0	-	1.000
					2.50	6256.2	1.177	0.095	1733.9	1.099	0.202	7990.1	1.160	0.096	23	I	0.984
					5.00	6098.7	1.183	0.096	2802.9	1.107	0.192	8901.6	1.159	0.101	37	I	0.971
					7.50	5978.0	1.182	0.097	3588.9	1.115	0.187	9566.9	1.157	0.106	47	I	0.959
					10.00	5878.9	1.184	0.099	4211.3	1.124	0.181	10090.3	1.159	0.110	55	I	0.950

(1)Other properties according to **Table 5.1**.
(2)FRP type 2 from **Table 5.1**.
(3)I: concrete crushing, II: debonding
(4)According to Equation 5.14 with β_T=3.5
(5)1.0 kip.in=0.113 kN.m

Table 5.7: Ultimate strength of the beams in Table 5.6

Set	ω_b	ω_s	ω_f	$(\omega_s+\omega_f)/\omega_b$	M_{ns} kip.in	M_{nf} kip.in	M_n (1) kip.in	Δ (%)	$Ø_{RC}$	$Ø_{NSM}/Ø_{RC}$ (2)	$Ø_{NSM}$	M_u (3) kip.in	M_u (4) kip.in
1	0.203	0.075	0.000	0.37	207.0	0.0	207.0	0	0.900	1.000	0.900	186.3	186.3
			0.016	0.45	204.3	54.2	258.5	26	0.900	1.000	0.900	232.7	225.3
			0.031	0.52	203.0	107.7	310.7	52	0.900	1.000	0.900	279.6	265.1
			0.047	0.60	201.6	160.8	362.4	78	0.900	1.000	0.900	326.2	304.5
			0.062	0.67	200.2	213.2	413.4	103	0.900	1.000	0.900	372.1	343.3
2	0.186	0.150	0.000	0.80	1577.2	0.0	1577.2	0	0.900	1.000	0.900	1419.5	1419.5
			0.051	1.08	1529.1	569.4	2098.5	33	0.900	0.997	0.897	1883.3	1811.8
			0.102	1.35	1497.9	922.4	2420.4	53	0.900	0.979	0.881	2133.1	2053.8
			0.152	**1.62**	**1473.7**	**1187.8**	**2661.5**	**69**	**0.900**	**0.952**	**0.857**	**2281.0**	**2235.0**
			0.203	1.89	1453.6	1402.6	2856.2	81	0.899	0.919	0.826	2360.2	2378.0
3	0.179	0.225	0.000	1.26	6497.3	0.0	6497.3	0	0.900	1.000	0.900	5847.6	5847.6
			0.102	1.82	6256.2	1733.9	7990.1	23	0.879	0.979	0.860	6872.0	6791.0
			0.203	2.39	6098.7	2802.9	8901.6	37	0.804	0.959	0.771	6859.6	6815.9
			0.305	2.96	5978.0	3588.9	9566.9	47	0.757	0.948	0.717	6860.0	6832.7
			0.407	3.53	5878.9	4211.3	10090.3	55	0.724	0.939	0.679	6852.9	6844.4

(1) $M_n = M_{ns} + M_{nf}$
(2) According to Equation 5.16
(3) $M_u = Ø_{NSM} M_n$: According to this study
(4) $M_u = Ø_{RC} (M_{ns} + 0.85 M_{nf})$: According to ACI 440.2R-08
1.0 kip.in=0.113 kN.m

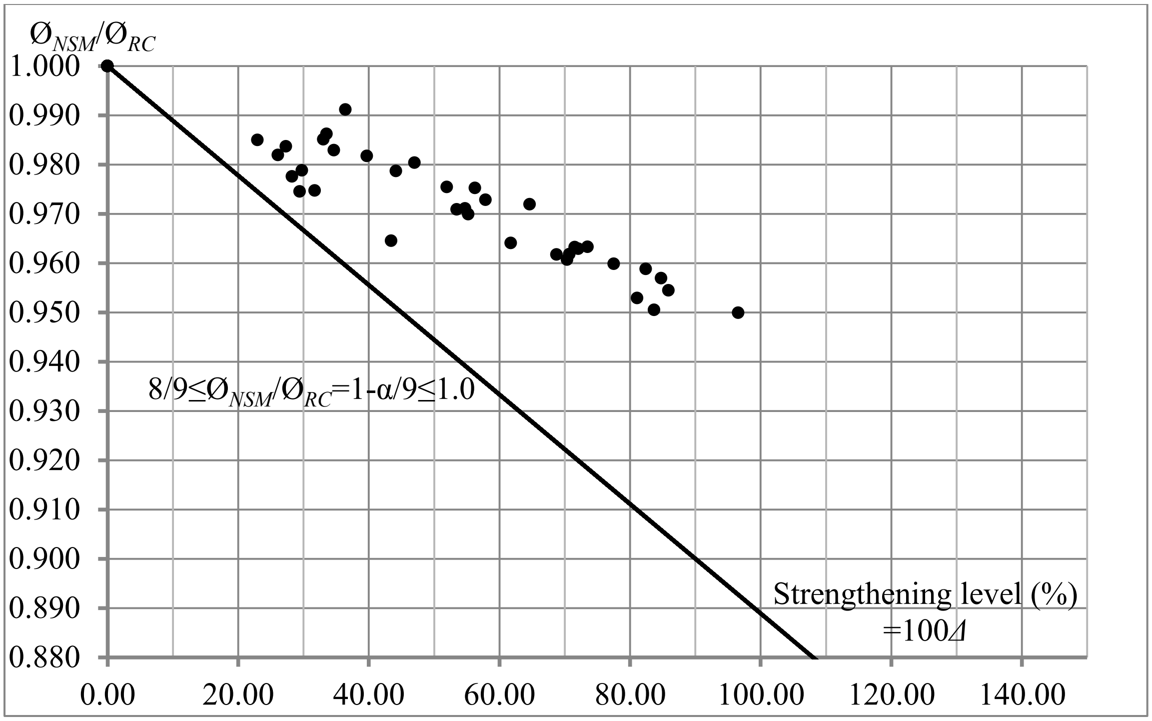

Figure 5.1: Calculated strength reduction factors for beams with $1.0 \le (\omega_s+\omega_f)/\omega_b \le 2.0$.

(Only points with $Ø_{NSM}/Ø_{RC} \le 1.0$ are shown.)

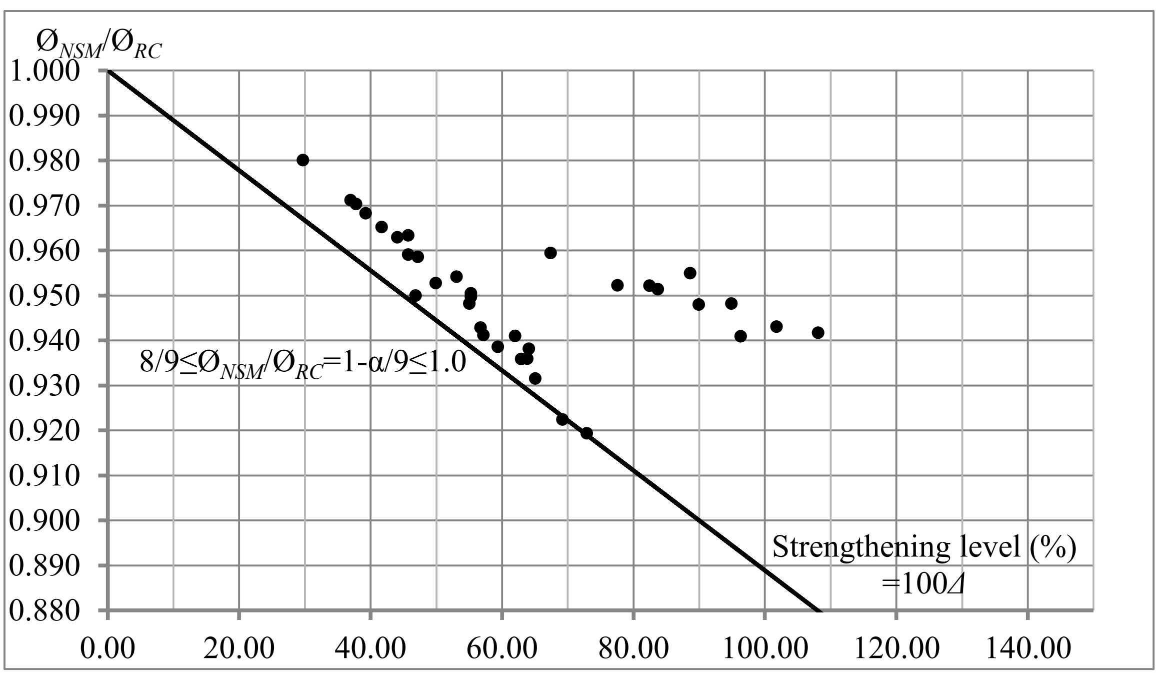

Figure 5.2: Calculated strength reduction factors for beams with $2.0 \le (\omega_s+\omega_f)/\omega_b \le 4.0$.

(Only points with $Ø_{NSM}/Ø_{RC} \le 1.0$ are shown.)

CHAPTER 6

6. CONCLUSIONS

The results and findings of this thesis are concisely recapitulated in **Table 6.1** and itemized as:

- Study I, analytically, formulates the live load factor as the function shown in the first row of Table 6.1. It also obtains approximate values for the life-time modification coefficient, κ.
- Study II, benefits from the numerical methods to find more accurate values of κ.
- Study III, introduces the "comparative reliability" concept and with that calculates new flexural strength reduction factors for beams and slabs internally reinforced with FRP bars (**Table 6.1**). It also concludes that Ø factor of 0.75, for shear, can be maintained if a stricter limit is imposed on maximum shear reinforcement with FRP stirrups.
- Study IV, obtains a flexural strength reduction factor that is exclusively calculated for NSM FRP bars, which is an enhancement to the current guideline that uses a Ø factor meant for steel RC flexural elements in combination with the ψ_f factor.

Further Investigation

The four studies that constitute this dissertation conform to a similar pattern of introducing a general methodology followed by its application to an especial case of interest. Other cases, however, can now be investigated taking advantage of the theoretical basis laid by these studies:

- The life-time of a structure may be incorporated into other time-dependent load cases such as wind and earthquake, using the methodology detailed by Studies I and II.
- Current North American design guidelines do not cover columns internally reinforced with FRP bars. The structural reliability of such members may be analyzed according to the procedures detailed in Studies III and IV in order to obtain their associated calibrated reduction factors.
- Study IV can be further advanced to include other types of external strengthening of RC members with FRP materials as well as other ultimate limit states.

Table 6.1: Summary of the results compared to the values in codes in practice

Study	Subject of study	Parameter of concern	According to ACI	According to this thesis
I and II	ACI 318-08 RC elements	Live load factor	$\gamma_L = 1.60$ [1]	$\gamma_{L_n} = 1.60\left[1+\kappa\ln\left(\frac{n}{50}\right)\right]$ [2]
III	ACI 440.1R-06 FRP RC elements	Strength reduction factors	Flexure, FRP rupture: Ø=0.55 [3]	Ø=0.70[4]
			Flexure, Concrete crushing:0.55 ≤Ø≤0.65 [3]	Ø=0.75[4]
			Shear: Ø=0.75 [5]	Ø=0.75[4]
		Maximum shear reinforcement	$V_f \leq 8\sqrt{f_c'}bd$ [6]	$V_f \leq 3V_c$[4]
IV	ACI 440.2R-08 strengthened RC elements	Strength reduction factor for flexural NSM systems	Overall: $0.65 \leq Ø_{NSM} = Ø_{RC} \leq 0.90$ [7] FRP: ψ_f=0.85 [8]	$8/9 \leq Ø_{NSM}/Ø_{RC} \leq 1.0$ [9] ψ_f is removed.

[1] ACI 318-08:9.2.1
[2] n is the expected life-time (years), κ is according to **Table 3.4.**
[3] ACI 440.1R-06:8.2.3
[4] Section 4.9
[5] ACI 440.1R-06:9.1.1
[6] ACI 440.1R-06:9.2.3
[7] ACI 440.2R-08:10.2.7 (Equation 5.22)
[8] ACI 440.2R-08:10.2.10(Equation 5.22)
[9] Equation 5.16

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APPENDIX A: STUDY I - DETAILS OF CALCULATION OF TARGET RELIABILITY INDEX

The nominal values in Equation 2.28 are related to their mean values by:

$$R_N = \frac{\mu_R}{\lambda_R} \quad \text{(A1)}$$

$$D = \frac{\mu_D}{\lambda_D} \quad \text{(A2)}$$

$$L = \frac{\mu_L}{\lambda_L} \quad \text{(A3)}$$

Equation A3 is the equivalent of Equation 2.20 for the special case of n=50. Equation 2.26 is formulated for a single load; therefore, the right-hand side of Equation 2.28 may be revised in terms of a single total load variable, Q, with a nominal value of Q_N, a load factor of γ and a bias factor of λ_Q, as Q is the sum of two independent variables Q_D, dead load, and Q_L, live load:

$$Q = Q_D + Q_L \quad \text{(A4)}$$

$$\mu_Q = \mu_D + \mu_L \quad \text{(A5)}$$

$$\sigma_Q^2 = \sigma_D^2 + \sigma_L^2 \quad \text{(A6)}$$

$$Q_N = D + L \quad \text{(A7)}$$

$$Q_N = \frac{\mu_Q}{\lambda_Q} \quad \text{(A8)}$$

$$\gamma Q_N = \gamma_D D + \gamma_L L \quad \text{(A9)}$$

$$\phi R_N = \gamma Q_N \quad \text{(A10)}$$

γ , λ_Q and δ_Q, depend on ρ , the ratio of live load to total load:

$$\rho = \frac{L}{D+L} = \frac{L}{Q_N} \quad \text{(A11)}$$

From Equations A7, A9 and A11 with γ_D=1.2 and γ_L=1.6, one obtains:

$$\gamma = (1-\rho)\gamma_D + \rho\gamma_L = 1.2 + 0.4\rho \tag{A12}$$

Similarly by substituting from Equations A1, A2 and A8 into Equation A5, it can be rewritten as:

$$\lambda_Q Q_N = \lambda_D D + \lambda_L L \tag{A13}$$

And therefore, substituting from Equation A11 and **Table 2.2**:

$$\lambda_Q = (1-\rho)\lambda_D + \rho\lambda_L = 1.05 - 0.05\rho \tag{A14}$$

Equation A6 can also be written as:

$$(\mu_Q\delta_Q)^2 = (\mu_D\delta_D)^2 + (\mu_L\delta_L)^2 \tag{A15}$$

Substituting bias factors and nominal values from Equations A2, A3 and A8 into Equation A15, one obtains:

$$(\lambda_Q Q_N \delta_Q)^2 = (\lambda_D D \delta_D)^2 + (\lambda_L L \delta_L)^2 \tag{A16}$$

Hence as, from Equation A11 and **Table 2.2**:

$$\lambda_Q\delta_Q = \sqrt{[(1-\rho)\lambda_D\delta_D]^2 + (\rho\lambda_L\delta_L)^2} = \sqrt{[0.105(1-\rho)]^2 + (0.18\rho)^2} \tag{A17}$$

With the factor and the probabilistic parameters of the total load calculated, the next step is to calculate the reliability index. The limit state function *G(Q,R)* can be written as:

$$G(Q,R) = R - Q \tag{A18}$$

It is more convenient for calculations to be carried out in terms of dimensionless variables. Therefore Equations A1, A8 and A10 are reworked as:

$$\frac{\mu_Q}{\phi\lambda_Q} = \frac{\mu_R}{\gamma\lambda_R} = \mu \tag{A19}$$

The dimensionless variables of resistance, *r*, and load, *q*, can be defined as:

$$q = \frac{Q}{\mu} \tag{A20}$$

$$r = \frac{R}{\mu} \tag{A21}$$

And their statistical parameters are:

$$\mu_q = \frac{\mu_Q}{\mu} = \phi\lambda_Q;\ \sigma_q = \frac{\sigma_R}{\mu} = \phi\lambda_Q\delta_Q;\ \delta_q = \delta_Q \tag{A22}$$

$$\mu_r = \frac{\mu_R}{\mu} = \gamma\lambda_R;\ \sigma_r = \frac{\sigma_R}{\mu} = \gamma\lambda_R\delta_R;\ \delta_r = \delta_R \tag{A23}$$

Equally, the limit state function can be defined in terms of q and r:

$$G(q, r) = r - q \tag{A24}$$

Using Equations A22 and A23, the reliability index can be expressed as:

$$\beta_{50} = \frac{\mu_r - \mu_q}{\sqrt{\sigma_r^2 + \sigma_q^2}} = \frac{\gamma\lambda_R - \phi\lambda_Q}{\sqrt{(\gamma\lambda_R\delta_R)^2 + (\phi\lambda_Q\delta_Q)^2}} \tag{A25}$$

By means of Equations A12, A14, A17 and A25, the reliability index can be calculated for any given ratio of live to total load for a 50-year life-span, which is regarded in this study as target reliability, β_T, for that loading condition.

APPENDIX B: STUDY I - DETAILS OF CALCULATION OF LIVE LOAD FACTOR

Similar to Equation A25, the reliability index for a life span of n years (β_n) can be obtained as:

$$\beta_n = \frac{\gamma_n \lambda_R - \phi \lambda_{Q_n}}{\sqrt{(\gamma_n \lambda_R \delta_R)^2 + (\phi \lambda_{Q_n} \delta_{Q_n})^2}} = \beta_{50} \tag{B1}$$

The time-dependent parameters, identified by subscript n can be calculated by generalizing their counterparts derived in **Appendix A**:

$$\gamma_n = (1-\rho)\gamma_D + \rho \gamma_{L_n} \tag{B2}$$

Which characterizes the general form of Equation A12. Similarly, time can be incorporated into Equations A14 and A17 as:

$$\lambda_{Q_n} = (1-\rho)\lambda_D + \rho \lambda_{L_n} \tag{B3}$$

$$\lambda_{Q_n}\delta_{Q_n} = \sqrt{[(1-\rho)\lambda_D\delta_D]^2 + \left(\rho \lambda_{L_n}\delta_{L_n}\right)^2} = \sqrt{[0.105(1-\rho)]^2 + (0.18\rho)^2} = \lambda_Q \delta_Q \tag{B4}$$

Noting that in Equation B3, λ_{L_n} is derived from Equation 2.22. Also, Equation B4 is simplified using Equations 2.24 and A17.

Although Equation B1 can now be solved for the exact value of γ_n, a simple approximation technique can be used to render it separable and obtain a less complicated final solution (Haldar and Mahadevan 2000) by introducing an intermediary variable, ε_n:

$$\varepsilon_n = \frac{\sqrt{(\gamma_n \lambda_R \delta_R)^2 + (\phi \lambda_{Q_n} \delta_{Q_n})^2}}{\gamma_n \lambda_R \delta_R + \phi \lambda_{Q_n} \delta_{Q_n}} \tag{B5}$$

Since all the parameters in Equation B5 are positive, then $\sqrt{2}/2 \leq \varepsilon_n \leq 1.00$. Also according to Equation B4, for a given ρ, the term $(\lambda_{Q_n}\delta_{Q_n})$ is constant, therefore within a small margin of error ε_n can be assumed to be constant and equal to its value at n=50:

$$\varepsilon_n \approx \varepsilon_{50} \tag{B6}$$

Based on Equations B5 and B6, Equation B1 can then be written as:

$$\beta_n = \frac{\gamma_n\lambda_R - \phi\lambda_{Qn}}{\varepsilon_{50}(\gamma_n\lambda_R\delta_R + \phi\lambda_{Qn}\delta_{Qn})} = \beta_{50} \tag{B7}$$

Which can be solved for γ_n as:

$$\gamma_n = \phi\frac{\lambda_{Qn}}{\lambda_R}\frac{1+\varepsilon_{50}\beta_{50}\delta_{Qn}}{1-\varepsilon_{50}\beta_{50}\delta_R} \tag{B8}$$

If n=50, the subscript n may be dropped and Equation B8 may be written simply as:

$$\gamma = \phi\frac{\lambda_Q}{\lambda_R}\frac{1+\varepsilon_{50}\beta_{50}\delta_Q}{1-\varepsilon_{50}\beta_{50}\delta_R} \tag{B9}$$

Equations B4, B8 and B9 result in:

$$\frac{\gamma_n}{\gamma} = \frac{\lambda_{Qn}}{\lambda_Q}\frac{1+\varepsilon_{50}\beta_{50}\delta_{Qn}}{1+\varepsilon_{50}\beta_{50}\delta_Q} = \frac{\lambda_{Qn}+\varepsilon_{50}\beta_{50}\lambda_Q\delta_Q}{\lambda_Q+\varepsilon_{50}\beta_{50}\lambda_Q\delta_Q} \tag{B10}$$

This can be solved for γ_{L_n} as:

$$\gamma_{L_n} = \gamma_L + 0.14\frac{\gamma}{\lambda_Q+\varepsilon_{50}\beta_{50}\lambda_Q\delta_Q}\ln\left(\frac{n}{50}\right) \tag{B11}$$

Which can be summarized as:

$$\gamma_{L_n} = 1.6\left[1+\kappa\ln\left(\frac{n}{50}\right)\right] \tag{B12}$$

$$\kappa = \frac{0.0875\gamma}{\lambda_Q+\varepsilon_{50}\beta_{50}\lambda_Q\delta_Q} \tag{B13}$$

κ is the “life-time modification coefficient”.

APPENDIX C: STUDY I - SIMPLIFIED METHOD FOR CALCULATION OF LIFE-TIME MODIFICATION COEFFICIENT

Equation B13 can be simplified by replacing the parameters ε_{50} and β_{50} with constant values:

The separation parameter, ε_{50}, can be approximated with sufficient accuracy, as 0.75.β_{50} may be replaced by β_T, the desirable level of reliability index i.e., for the limit states of **Table 4**: β_T=3.5 for beams, β_T=2.5 for slabs and β_T=4.0 according to Novak and Szerszen (2003). Let β_T=4.0 and ε_{50}=0.75, **Fig. C1** shows κ as a function of ρ, which for normal loading cases ($0.5 \leq L/D \leq 2.0$) averages at 0.09, with negligible deviation. Thus it is reasonable to substitute κ in Equation 2.30 with the unique value of 0.09 to cover all the ultimate states and live load ratios (Equation 2.31).

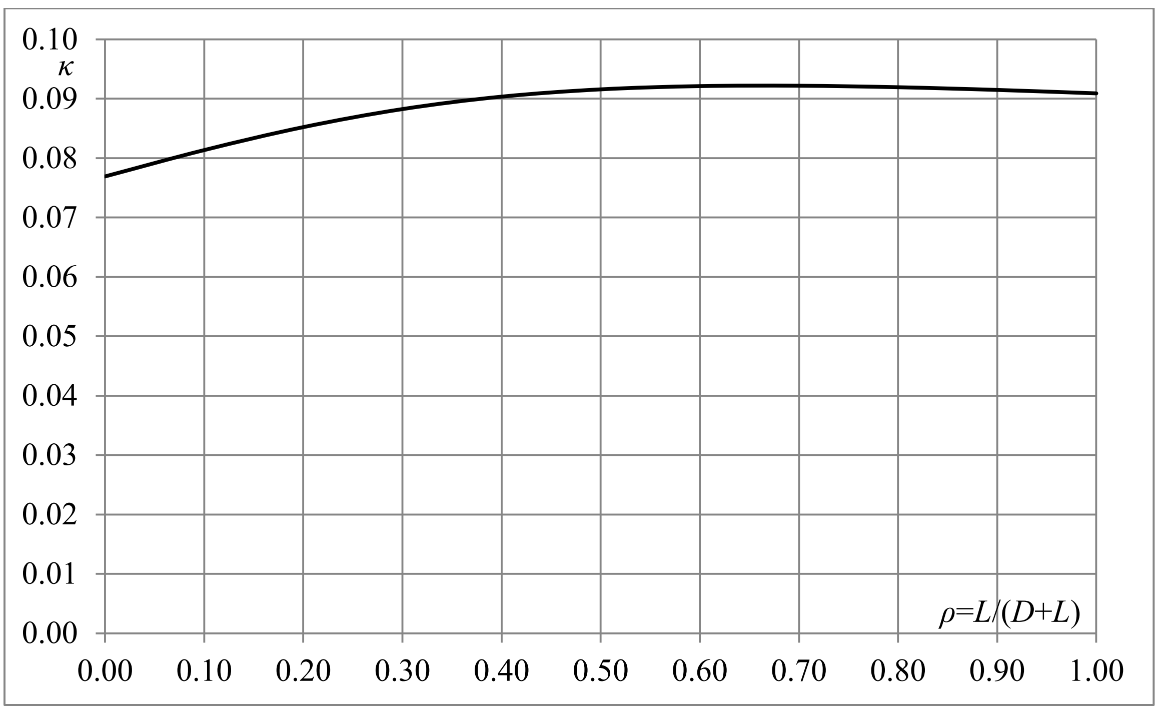

Figure C1- life-time modification coefficient (κ) vs. live load ratio (β_T=4.0 and ε_{50}=0.75)

APPENDIX D: STUDY II - RACKWITZ-FIESSLER METHOD

The Rackwitz-Fiessler method is an iterative procedure whose cycles consist of two major steps: first, finding the "design point"; and, second, calculating the "equivalent normal" values of the mean and standard deviation for each non-normal variable. The input data of the Rackwitz-Fiessler are the "real" mean and standard deviation of the non-normal variables of the limit state function, $G(Q,R)=R-Q$. R is the resistance and load, Q, as assumed earlier, is constituted of dead and live load components, Q_D and Q_L (For a complete list of symbols see **NOTATIONS**). For the ease of calculations, these variables may be replaced by their dimensionless counterparts:

$$q = d + l \tag{D1}$$

$$q = \frac{Q}{\mu};\ d = \frac{Q_D}{\mu};\ l = \frac{Q_L}{\mu} \tag{D2}$$

Where μ is defined as:

$$\mu = \frac{\mu_Q}{\phi\lambda_Q} = \frac{Q_N}{\phi} \tag{D3}$$

ρ stands for the ratio of design live load, L, to total load, Q_N, or:

$$\rho = \frac{L}{D+L} = \frac{L}{Q_N} \tag{D4}$$

Which in combination with Equation D3 results in:

$$\mu = \frac{D}{(1-\rho)\phi} = \frac{L}{\rho\phi} \tag{D5}$$

Replacing the nominal values of loads by their means, using the definition of bias factor, it can be obtained that:

$$\mu = \frac{\mu_D}{(1-\rho)\phi\lambda_D} = \frac{\mu_L}{\rho\phi\lambda_L} \tag{D6}$$

Hence the statistical parameters of d, l and q can be calculated as:

$$\mu_d = \frac{\mu_D}{\mu} = (1-\rho)\phi\lambda_D;\ \sigma_d = \frac{\sigma_D}{\mu} = (1-\rho)\phi\lambda_D\delta_D;\ \delta_d = \delta_D \tag{D7}$$

$$\mu_l = \frac{\mu_L}{\mu} = \rho\phi\lambda_L; \sigma_l = \frac{\sigma_L}{\mu} = \rho\phi\lambda_L\delta_L;\ \delta_l = \delta_L \quad \text{(D8)}$$

$$\mu_q = \mu_d + \mu_l \quad \text{(D9)}$$

$$\sigma_q^2 = \sigma_d^2 + \sigma_l^2 \quad \text{(D10)}$$

The statistical parameters of r, the dimensionless form of resistance, R, are taken from **Appendix A** (Equation A23) and repeated here:

$$\mu_r = \frac{\mu_R}{\mu} = \gamma\lambda_R;\ \sigma_r = \frac{\sigma_R}{\mu} = \gamma\lambda_R\delta_R;\ \delta_r = \delta_R \quad \text{(D11)}$$

With the input data calculated the details of the two aforesaid iterative steps of the method are as follows:

<u>First Step: Design point</u>

The design point, $x^*(q^*, r^*)$, can be defined as the closest point of the function $G(q, r)=r-q=0$ to the origin, when G is formulated in terms of reduced variables of load and resistance, z_q and z_r.

$$z_q = \frac{q-\mu_q}{\sigma_q} \quad \text{(D12)}$$

$$z_r = \frac{r-\mu_r}{\sigma_r} \quad \text{(D13)}$$

z_q and z_r possess a mean of 0 and a standard deviation of 1. This shortest distance (i.e., between the design point and origin) is then equal to the reliability index, β (**Fig. D1**). For an arbitrary limit state function, the general approach to this step is the implementation of the Lagrange multipliers method and numerical or trial-and-error solution of the resultant equations (Nowak and Collins 2000). In this study, that deals with a linear limit state function, an alternative and more conceptual solution is presented that circumvents the Lagrange multipliers and yields closed-form solutions:

$$G(z_q, z_r) = r - q = (\mu_r - \mu_q) + z_r\sigma_r - z_q\sigma_q = 0 \quad \text{(D14)}$$

From **Fig. A1** it can be concluded that:

$$G(z_q^*, z_r^*) = (\mu_r - \mu_q) + z_r^*\sigma_r - z_q^*\sigma_q = 0 \tag{D15}$$

$$(z_q^*)^2 + (z_r^*)^2 = \beta^2 \tag{D16}$$

While the reliability index, β, is conventionally defined as:

$$\beta = \frac{\mu_r - \mu_q}{\sqrt{\sigma_r^2 + \sigma_q^2}} \tag{D17}$$

By solving Equations D15, D16 and D17 the reduced design point can be calculated as:

$$z_q^* = \frac{\sigma_q(\mu_r - \mu_q)}{\sigma_r^2 + \sigma_q^2} \tag{D18}$$

$$z_r^* = \frac{-\sigma_r(\mu_r - \mu_q)}{\sigma_r^2 + \sigma_q^2} \tag{D19}$$

Mapping (z_q^*, z_r^*) into (q, r) coordinates using Equations D12 and D13, the design point (q^*, r^*) is found as:

$$q^* = r^* = \frac{\mu_r\sigma_q^2 + \mu_q\sigma_r^2}{\sigma_q^2 + \sigma_r^2} \tag{D20}$$

q^* must be decoupled to its two components (d^*, l^*). To this end, the limit state function $G(d, l, r)$=0 that describes a plane in the (d, l, r) space must be reformulated in terms of their reduced variables, (z_d, z_l, z_r):

$$G(d, l, r) = r - (d + l) = 0 \tag{D21}$$

$$z_d = \frac{d - \mu_d}{\sigma_d} \tag{D22}$$

$$z_l = \frac{l - \mu_l}{\sigma_l} \tag{D23}$$

The reduced form of r is calculated from Equation D13. Substituting these reduced forms into the limit state function (Equation D21) and simplifying it with Equation D9, the limit state plane in the (z_d, z_l, z_r) space is describable as:

$$G(z_d, z_l, z_r) = (\mu_r - \mu_q) + z_r\sigma_r - z_l\sigma_l - z_d\sigma_d = 0 \tag{D24}$$

As mentioned earlier, by definition, the closest point of this plane to the origin is the reduced design point, $z^*(z_d^*, z_l^*, z_r^*)$, while its distance from the origin is equal to the reliability index, β:

$$(z_d^*)^2 + (z_l^*)^2 + (z_r^*)^2 = \beta^2 \quad \text{(D25)}$$

It can be derived from Equations D16 and D25 that:

$$(z_d^*)^2 + (z_l^*)^2 = (z_q^*)^2 \quad \text{(D26)}$$

Equation D1 can be expressed in terms of reduced variables as well:

$$\mu_q + z_q\sigma_q = (\mu_d + z_d\sigma_d) + (\mu_l + z_l\sigma_l) \quad \text{(D27)}$$

Combined with Equation D9, Equation D27 may be simplified as:

$$z_q\sigma_q = z_d\sigma_d + z_l\sigma_l \quad \text{(D28)}$$

Naturally, Equation D28 must be satisfied at $z_q^*(z_d^*, z_l^*)$ as well:

$$z_q^*\sigma_q = z_d^*\sigma_d + z_l^*\sigma_l \quad \text{(D29)}$$

Where z_q^* can be obtained from Equation D18.The two conditions stated by Equations D26 and D29 may have a geometrical interpretation similar to that of the definition of reliability index and design point shown in **Fig. D1**, which is plotted in **Fig. D2**. Substituting from Equation D10 into the two Equations D26 and D29 they can be solved for z_d^* and z_l^* as:

$$z_d^* = \frac{\sigma_d}{\sigma_q} z_q^* = \frac{\sigma_d(\mu_r - \mu_q)}{\sigma_r^2 + \sigma_q^2} \quad \text{(D30)}$$

$$z_l^* = \frac{\sigma_l}{\sigma_q} z_q^* = \frac{\sigma_l(\mu_r - \mu_q)}{\sigma_r^2 + \sigma_q^2} \quad \text{(D31)}$$

By substituting values of z_d^* and z_l^* into Equations D22 and D23, the load components of the design point are finally calculated as:

$$d^* = \mu_d + \frac{\sigma_d^2}{\sigma_r^2 + \sigma_q^2}(\mu_r - \mu_q) \quad \text{(D32)}$$

$$l^* = \mu_l + \frac{\sigma_l^2}{\sigma_r^2+\sigma_q^2}(\mu_r - \mu_q) \tag{D33}$$

It should be noted that:

$$r^* = d^* + l^* \tag{D34}$$

Which conforms to Equation D20.

Second Step: Equivalent normal parameters

This step requires the replacement of the mean and standard deviation, μ_X and σ_X, of a non-normal variable X of the limit state function, G, with "equivalent normal parameters" $\mu^e{}_X$ and $\sigma^e{}_X$, that possess the same values of PDF and CDF at the design point x^*. X can be any of the live load, l, or resistance, r, variables. This can be translated into mathematical terms as (Nowak and Collins 2000):

$$\sigma_X^e = \frac{1}{f_X(x^*)}\phi\left[\Phi^{-1}\left(F_X(x^*)\right)\right] \tag{D35}$$

$$\mu_X^e = x^* - \sigma_X^e\left[\Phi^{-1}\left(F_X(x^*)\right)\right] \tag{D36}$$

Where ϕ is the PDF for the standard normal distribution and Φ^{-1} is the inverse of the CDF for the standard normal distribution. F_X and f_X indicate the CDF and PDF of the variable X respectively. For the EVD Type I load variable of l, the equivalent parameters can be calculated by substituting its CDF and PDF (Study I) into Equations D35 and D36. If arbitrary-point-in-time live load is of concern, probability functions of the gamma distribution are to be used:

$$f_X(x) = x^{k-1}\frac{e^{-\frac{x}{\theta}}}{\theta^k\Gamma(k)} \tag{D37}$$

$$\mu_X = k\theta \tag{D38}$$

$$\sigma_X^2 = k\theta^2 \tag{D39}$$

For the lognormal variable, r, or resistance, Equations D35 and D36 can be further simplified as (Nowak and Collins 2000):

$$\sigma_r^e = r^* \sigma_{\ln(r)} \tag{D40}$$

$$\mu_r^e = r^* \left[1 - \ln(r^*) + \mu_{\ln(r)}\right] \tag{D41}$$

Where:

$$\sigma_{\ln(r)}^2 = \ln(1 + \delta_r^2) \tag{D42}$$

$$\mu_{\ln(r)} = \ln(\mu_r) - \frac{\sigma_{\ln(r)}^2}{2} \tag{D43}$$

<u>Iteration</u>

When the equivalent parameters are calculated, step 1 must be repeated anew, using the equivalent values to find a new design point, from which new equivalent parameters are calculated. This cycle must be repeated until the desirable convergence is achieved. The following recapitulates the procedure:

1. For any given value of ρ, the "real" statistical parameters of load and resistance are calculated from Equations D7 to D11.
2. The design point is calculated from Equations D32 to D34.
3. The reliability index is calculated from Equation D17.
4. The equivalent parameters of live load and resistance are calculated from Equations D35 and D36 for live load and D40 and D41 for resistance. Dead load is already a normal variable and needs no alteration in its parameters.
5. Steps (b) to (d) are repeated, with the new equivalent parameters replacing the old ones (from the previous cycle) until both β and design point converge.

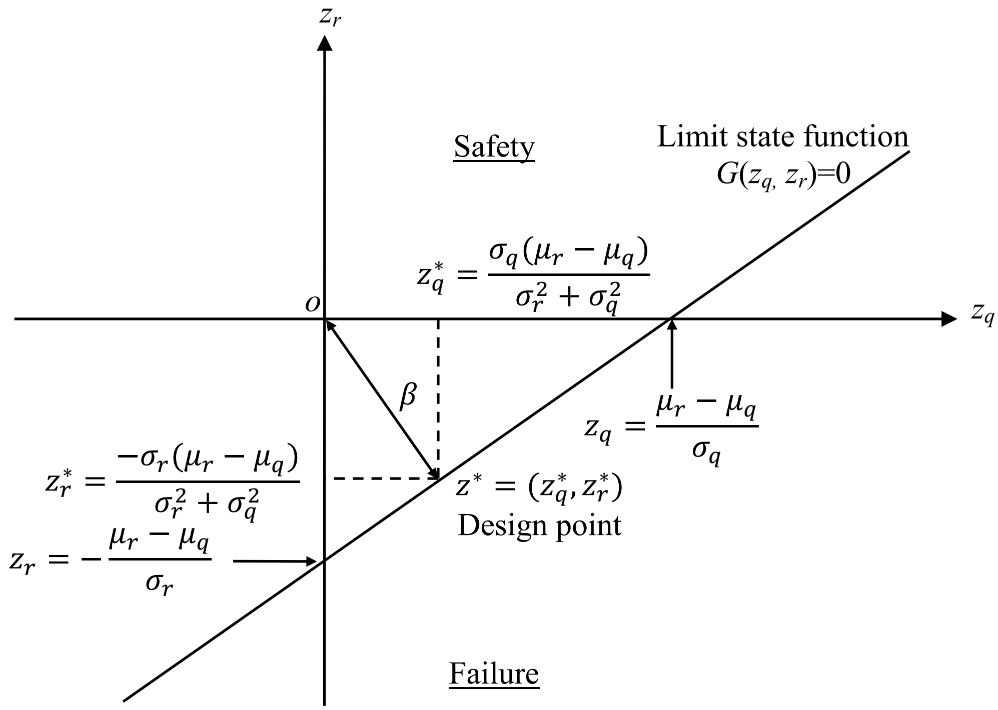

Figure D1- Definition of reliability index and design point in the space of reduced variables

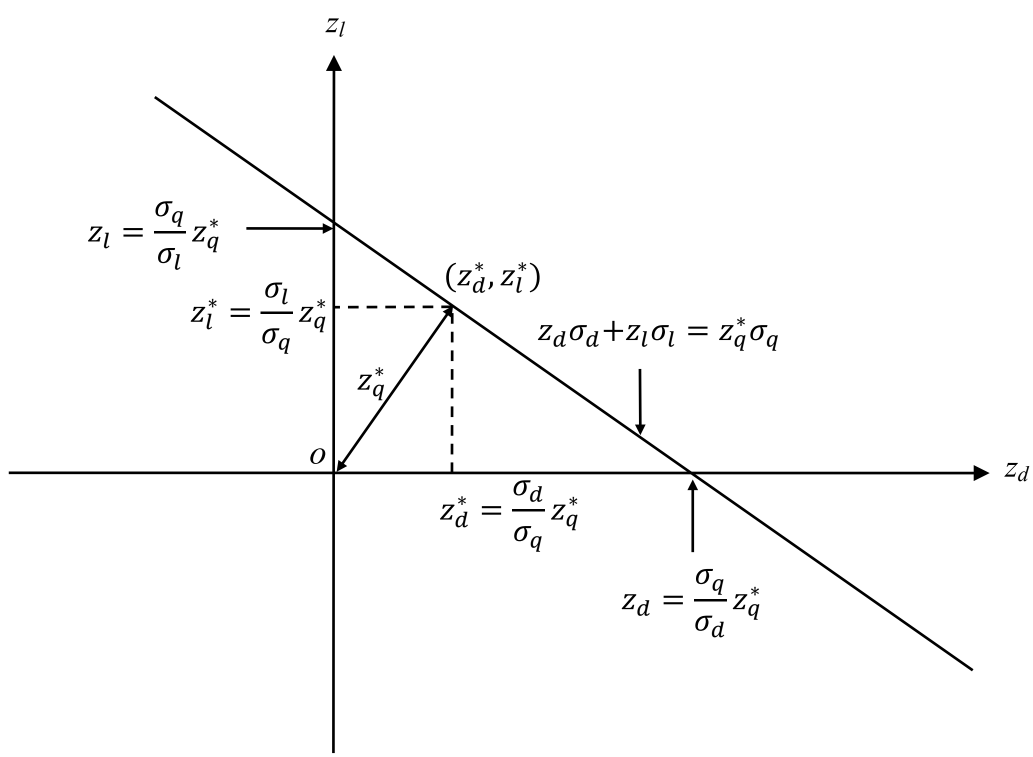

Figure D2- Geometrical display of reduced components of load at the design point

APPENDIX E: STUDY II - MONTE CARLO SIMULATION

The general procedure for generating random samples, x_i, of a variable X with an arbitrary distribution of $F_X(x)$ may be described by (Nowak and Collins 2000):

$$x_i = F_X^{-1}(u_i) \tag{E1}$$

Where u_i is a sample of a uniformly distributed variable between 0 and 1, and F_X^{-1} is the inverse of F_X. For each set of randomly generated samples of load and resistance, (d_i, l_i, r_i), the limit state function, G_i, is calculated and the probability of failure is estimated as:

$$P = \frac{N_f}{N} \tag{E2}$$

Where N_f is the number of failures observed (i.e., events of $G_i<0$) and N is the total number of simulations or the sets of random samples. The reliability index follows the conventional definition:

$$\beta = -\Phi^{-1}(P) \tag{E3}$$

Φ^{-1} is the inverse of the cumulative distribution function (CDF) for the standard normal distribution.

The accuracy of the probability estimates, needless to say, depends heavily on the number of simulations. To assess this accuracy, it should be noted that the estimated probability, P, is a random variable itself whose mean, μ_P, and coefficient of variation, δ_P, are related to the theoretically correct probability, P_{true}, by (Nowak and Collins 2000):

$$\mu_P = P_{true};\ \delta_P = \sqrt{\frac{1-P_{true}}{N(P_{true})}} \tag{E4}$$

Knowing that P_{true}, although unknown, is relatively small and assuming that the sample size, N, is large enough so that $P \approx P_{true}$, Equations E2 and E4 can be combined as:

$$\delta_P \approx \frac{1}{\sqrt{N_f}} \quad \text{(E5)}$$

Which is used as the indicator of accuracy in this study. To calculate the target reliability indices for cases similar to those presented in **Table 3.1**, each simulation is repeated until 400 events of failure are recorded (N_f=400) which corresponds to a variation of 5.0 percent (δ_P=0.05), a variation deemed small enough to justify the use of the outcomes as credible pointers towards the precision of calculations. The total number of required simulations, hence, varies approximately from $N=4\times10^4$ (if $P=1\times10^{-2}$ or $\beta\approx2.3$) to $N=4\times10^{12}$ (if $P=1\times10^{-10}$ or $\beta\approx6.4$), certainly increasing as the probability of failure decreases or equally the reliability index increases.

APPENDIX F: STUDY III - CALCULATION OF NOMINAL SHEAR STRENGTH OF BEAMS AND THEIR STATISTICAL PARAMETERS

The nominal shear capacity, V_n, is calculated according to Chapter 9 of ACI 440.1R-08.

$$V_n = V_c + V_f \tag{F1}$$

Where V_c and V_f are the contributions of concrete and FRP stirrups respectively:

$$V_c = \left(\frac{5}{2}\right) 2k\sqrt{f_c'}bd_f \tag{F2}$$

Where f'_c is the compressive strength of concrete, b is the width of the beam, d is the effective width and k is the ratio of depth of neutral axis to reinforcement depth, d:

$$k_c = \sqrt{2(\rho_f n_f) + (\rho_f n_f)^2} - (\rho_f n_f) \tag{F3}$$

Where ρ_f is the ratio of the longitudinal FRP bars and n_f is the modular ratio:

$$n_f = \frac{E_f}{E_c} \tag{F4}$$

E_f and E_c are the modulus of elasticity of FRP and concrete.

$$V_f = \frac{A_{fv} f_{fv} d_f}{s} \tag{F5}$$

A_{fv} is the area of FRP stirrups within a spacing of s. The tensile strength of FRP for shear design, f_{fv}, is calculated as:

$$f_{fv} = 0.004E_f \leq f_{fb} \tag{F6}$$

Where E_f is the modulus of elasticity of stirrups. f_{fb}, strength of bent portion of FRP stirrups, depends on that r_b/d_b, the ratio of internal radius of bend in stirrups to their diameter which here is assumed to be equal to 3, the minimum recommended by the guideline.

$$f_{fb} = \left(0.05\frac{r_b}{d_b} + 0.3\right) f_{fu} = 0.45 f_{fu} \leq f_{fu} \tag{F7}$$

f_{fu} is the ultimate longitudinal tensile strength of FRP stirrups. Examples of calculations are presented in **Tables F1** and **F2**. The first, from Yost et al. (2001), exemplifies beams without shear reinforcement while the second, from Nagasaka et al. (1993), represents shear reinforced beams. As in the second case V_f exceeds $3V_c$, the nominal strength, V_n, is calculated and shown twice in **Table F1**: first, as the sum of the two components of shear strength and second, according to the modification suggested by Equations 4.31 and 4.32. Such cases of "excessive" FRP contribution are disregarded when the aim is calculating the statistical parameters of shear resistance under the restrictions of Equations 4.31 and 4.32. When bias factors are calculated for every case (beams with and without stirrups are treated separately), the overall bias factor and coefficient of variation for resistance are calculated as, respectively, the mean and coefficient of variation of individual bias factors (**Table 4.7**).

Table F1: Examples of properties of specimens

				FRP bars		FRP stirrups			
Reference	f'_c ksi (MPa)	b in. (mm)	d_f in. (mm)	E_f ksi (GPa)	A_f in.^{2} (mm^2)	E_f ksi (GPa)	A_{fv} in.^{2} (mm)	f_{fu} ksi (Mpa)	s in. (mm)
Yost et al. (2001)	5.3 (36.3)	9.0 (229)	11.25 (286)	5800 (40)	1.116 (720)	-	0	-	-
Nagasaka et al. (1993)	3.3 (23)	9.84 (250)	9.96 (253)	8120 (56)	1.860 (1200)	8990 (62)	0.017 (100)	121.8 (840)	1.57 (40)

Table F2: Examples of calculations for beams of Table F1

Reference	ρ_f (%)	n_f	k_c	f_{fv} ksi(Mpa)	f_{fb} ksi(Mpa)	V_c kips (kN)	V_f kips (kN)	V_f/V_c	V_n kips (kN)	V_{exp} kips (kN)	**Bias** (λ_R)
Yost et al. (2001)	1.10	1.41	0.162	-	-	4.5 (20.1)	-	0	4.5 (20.1)	8.8 (39.1)	1.95
Nagasaka et al. (1993)	1.90	2.46	0.262	36.0 (248)	54.8 (378)	7.1 (31.8)	35.3 (156.9)	4.97	42.4 (188.7)	43.6 (194.0)	1.03
Nagasaka et al. (1993)(1)	1.90	2.46	0.262	36.0 (248)	54.8 (378)	7.1 (31.8)	35.3 (156.9) (1)	4.97(1)	28.6 (127.2) (1)	43.6 (194.0)	-(1)

(1)Modified method: $V_f \geq 3V_c$: $V_n = 4V_c$. Case is neglected.

APPENDIX G: STUDY IV - EXAMPLE OF SIMULATION TECHNIQUE

The general procedure for generating random samples, x_i, of a lognormal variable X with a coefficient of variation of $\delta_X \leq 0.20$, may be described as (Nowak and Collins 2000):

$$x_i = \mu_X e^{z_i \delta_X} \tag{G1}$$

$$z_i = \Phi^{-1}(u_i) \tag{G2}$$

Where u_i is a sample of a uniformly distributed variable between 0 and 1 and $\Phi^{-1}$is the inverse of the standard normal cumulative distribution. κ_m is assumed to be uniformly distributed, therefore its random sample is generated by a simple interpolation over its range of distribution ($0.60 \leq \kappa_m \leq 0.90$):

$$\kappa_{mi} = 0.60 + 0.30u_i \tag{G3}$$

The table below shows an example of a simulation for an NSM strengthened slab (**Table 5.5**, Set 2, second row):

Item	**Nominal Value (*N*)**	**Bias (*λ*)**	**CoV (*δ*)**	**Mean (*μ*= *λN*)**	u_i	z_i	x_i
f'_c ksi (MPa)	4.0 (27.6)	1.24	0.10	4.96 (34.2)	0.766	0.726	5.33(36.8)
f_y ksi (MPa)	60 (414)	1.145	0.05	68.7 (474)	0.539	0.098	69.0 (476)
f_{fu} ksi (MPa)	250 (1725)	1.20	0.08	300 (2070)	0.630	0.332	308.1(2126)
E_f ksi (GPa)	20000(138)	1.04	0.08	20800 (143)	0.544	0.111	20984(145)
κ_m	0.70	1.071	0.115	0.75	0.663	-	0.799
d_s in.(mm)	6.5(165)	0.92	0.12	5.98 (152)	0.757	0.697	6.50 (165)
c in.(mm)	1.5(38)	0.92	0.12	1.38 (35)	0.513	0.033	1.39 (35)
d_f in.(mm)	8.0(203)	-	-	7.36 (187)	-	-	7.89 (200)
b in.(mm)	12.0(305)	1.00	0.00	12.00 (305)	-	-	12.00 (305)
A_s in.^{2}(mm^2)	0.288(186)	1.00	0.015	0.288 (184)	0.227	-0.749	0.285 (184)
A_f in.^{2}(mm^2)	0.02(12.9)	1.00	0.03	0.02 (12.9)	0.703	0.533	0.020 (12.9)
P_s	1.00	1.02	0.06	1.02	0.330	-0.440	0.993
P_f	1.00	1.00	0.06	1.00	0.682	0.473	1.029

Excluding the last the two rows, the nominal values of flexural contribution can be calculated from the data in the second column, using ACI 440.2R-08 design guide:

M_{ns}=106.76 kip.in (12.06 kN.m), M_{nf}=26.88 kip.in (3.04 kN.m), $M_n=M_{ns}+M_{nf}$=133.64 kip.in (15.1 kN.m)

These contributions for the *i*th random sample, denoted by subindex *i*, may be calculated from the same data in the last column:

M_{si}=122.72 kip.in (13.86 kN.m), M_{fi}=37.57 kip.in (4.24 kN.m), $M_i=M_{si}+M_{fi}$ =160.29 kip.in. (18.10 kN.m)

Eventually, applying the professional factors from the last two rows, the random samples of resistance are obtained:

$R_{si}=P_{si}.M_{si}$=121.86 kip.in(13.77 kN.m), $R_{fi}= P_{fi}.M_{fi}$ =38.66 (4.37 kN.m), $R_i=R_{si}+R_{fi}$=160.52 kip.in. (18.14 kN.m)

R_i is one random sample of the flexural strength of a member defined by the nominal or deign values given in the table. The statistical parameters of *R* can be calculated from the samples, once the number of simulations is large enough to represent the population. This number can be decided on by repeating the simulation and measuring the consistency of the outcomes. **Fig. G1** portrays how the sample size is determined in this study. For the slab in the example, the simulation is repeated 10 times, each time with *n* samples (*n*=1,10,100,…) resulting in 10 different values of mean strength , μ_R, for each sample size, *n*. Let μ_S and σ_S represent the mean and standard deviation of the 10 values of μ_R for any given *n*. **Fig. G1** shows how, as expected, the variation of the results declines as the sample size grows, so that by *N*=10,000 the outcome is virtually deterministic. To eliminate any reservation, in this study each simulation contains 20,000 samples.

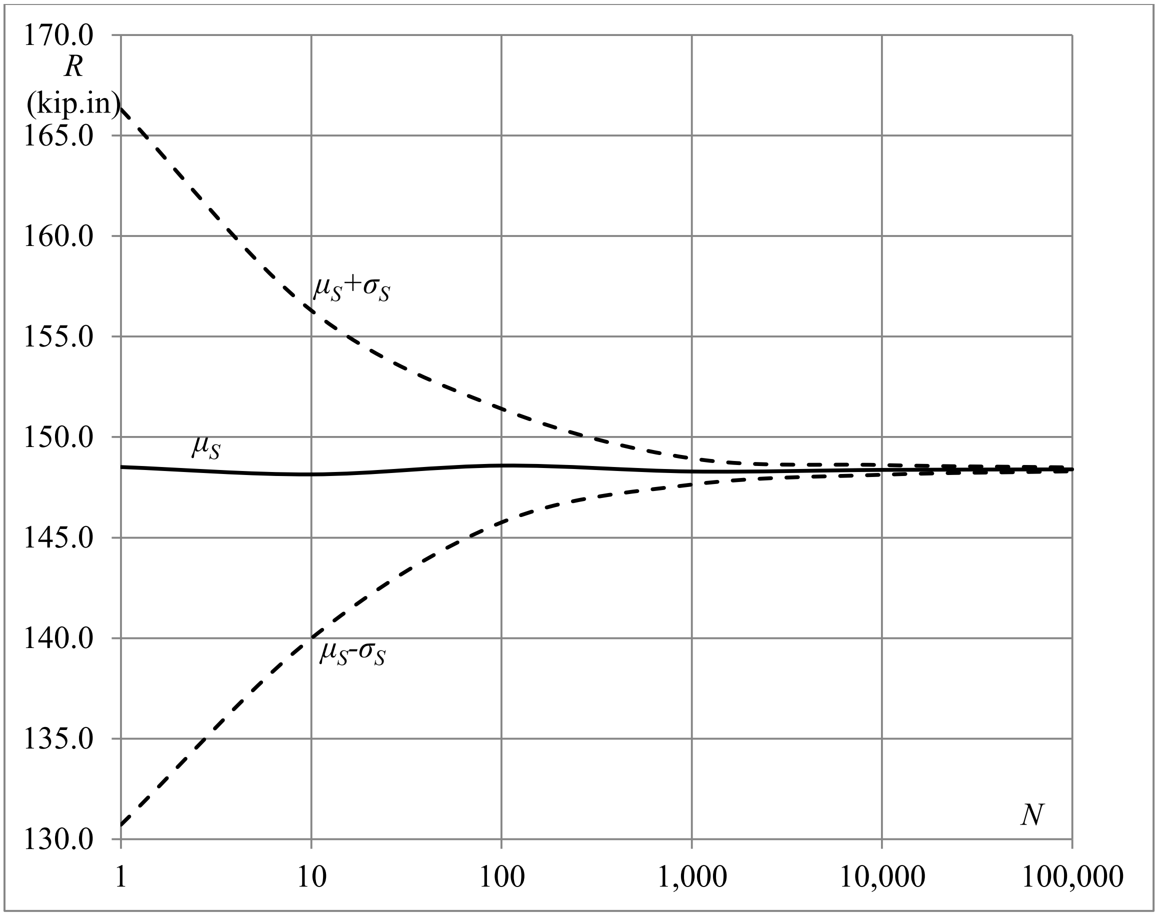

Figure G1: Variation of calculated mean resistance vs. number of samples for the slab in example (1.0 kip.in=0.113 kN.m).